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SEARCH-AND-MATCHING FRICTIONS AND LABOUR MARKET DYNAMICS IN LATVIA



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ABBREVIATIONS

- AOB alternating-offer wage bargaining
- APP annualised percentage points
- AR(1) first order autoregression
- BGG Bernanke, Gertler and Gilchrist (1999)
- CET Christiano, Eichenbaum and Trabandt (2013)
- CPI consumer price index
- CTW Christiano, Trabandt and Walentin (2011)
- dHRW den Haan, Ramey and Watson (2000)
- DM Diebold–Mariano
- DSGE dynamic stochastic general equilibrium
- EHL Erceg, Henderson and Levin (2000) FOC first order condition
- GDP gross domestic product
- HPD highest posterior density
- i.d.d. independent and identically distributed
- IRF impulse response function
- MAE mean absolute deviation
- MDD marginal data density
- MEI marginal efficiency of investment
- MPL marginal product of labour
- RMSE root mean squared error SVAR - structural vector autoregression
- US United States

ABSTRACT

This paper examines, in an estimated, full-fledged New Keynesian DSGE model with Nash wage bargaining, sticky wage and high value of leisure akin to Christiano, Trabandt and Walentin (2011), whether search-and-matching frictions in the labour market can explain aggregate labour market dynamics in Latvia. If vacancies are not observed, the model can, to a reasonable degree, generate realistic variance and dynamics of unemployment and the correlation between unemployment and (latent) vacancies, yet at the expense of too volatile vacancies. As a by-product, one quarter ahead forecasts of hours worked and GDP exhibit less excess volatility and, thus, are more precise compared to a model without search-and-matching frictions. However, if both unemployment and vacancies are observed and a shock to matching efficiency is allowed for, then cyclical behaviour of forecasted vacancies as well as correlation between unemployment and vacancies tend to counter the data (to the advantage of a better fit of vacancy volatility), and the smoothed matching efficiency is counter-intuitively counter-cyclical. Hence the model cannot fit the three statistics - variance of unemployment and vacancies, and correlation between the two, simultaneously.

Keywords: DSGE model, unemployment, small open economy, Bayesian estimation, currency union, forecasting

JEL codes: E0, E3, F0, F4, G0, G1

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The author is an employee of the Monetary Policy Department of Latvijas Banka. This report is released to inform interested parties of the research and to encourage discussion. The views expressed in this paper are those of the author and do not necessarily reflect the views of Latvijas Banka.

NON-TECHNICAL SUMMARY

The standard business cycle approach of modelling labour markets without explicit unemployment (as in Erceg, Henderson and Levin (2000); hereinafter, EHL) has its drawbacks. Its main drawback is that it has no implications for unemployment (socalled extensive margin of labour supply) and, thus, the variation of total hours worked is attributed solely to the variation in hours per employee (intensive margin). Second, it also tends to induce too little persistence in hours worked (see, e.g. Buss (2015)).

In reality, much of the variation in total hours worked is generated by the extensive margin of labour supply. To quantify it, this paper applies simple data variance decomposition to the Latvian data for the period of Q1 2002–Q4 2012 as detailed in the next Section. Though the data are noisy and, thus, the decomposition is rough, according to it, more than a half of the variation in total hours worked is explained by the variation in the number of employees. The two employment margins have different economic policy implications, and, thus, it is useful to distinguish between the two in economic analysis.

The search-and-matching theory is the most widespread economic theory of the labour market. However, much of the lengthy literature on DSGE models with search-and-matching frictions in the labour market is devoted to either calibrated models or studies of the US data. Rarely models are estimated using non-US data, and even more so using a full-fledged model. This paper adds to the literature by studying the ability of a richly specified New Keynesian DSGE model with searchand-matching frictions to fit the key moments of unemployment and vacancies, particularly for a non-US country. The model in this paper is closest to Christiano, Trabandt and Walentin ((2011); hereinafter, CTW). The innovation of this paper compared to CTW is that it 1) adjusts the model to a member country of a currency union, 2) estimates the model for Latvia, and most importantly 3) studies the model's ability to fit both unemployment and vacancies simultaneously, not in isolation. This is done by using two specifications of the model: in one of them, the vacancies data are unobserved and the matching function is calibrated, resembling the CTW specification but with implications for (latent) vacancies; in the other specification, the vacancies data are observed and the matching function, including the shock to the matching efficiency, is estimated.

This paper confirms CTW findings that the model can fit unemployment well. But the paper goes a step further and finds that CTW-favoured specification can fit also the correlation between unemployment and vacancies rather well; however, the decent fit of the above two statistics comes at a high cost of having vacancy standard deviation multiple (specifically, 2.9) times higher than in the data.

However, if both unemployment and vacancies are observed and a shock to the matching efficiency is allowed for, then the cyclical behaviour of forecasted vacancies as well as the correlation between unemployment and vacancies tend to counter the data (to the advantage of a better fit of vacancy volatility), and the smoothed matching efficiency is counter-intuitively counter-cyclical. Hence the model cannot fit the three statistics – variance of unemployment and vacancies, and correlation between the two, simultaneously.

As a by-product of adding search-and-matching frictions to the model, one quarter ahead forecasts of hours worked and GDP exhibit less excess volatility and, thus, are more precise compared to a model without search-and-matching frictions.

There are few studies that use estimated, full-fledged DSGE models with searchand-matching frictions and study the fit of the data moments. Christiano, Eichenbaum and Trabandt ((2013); hereinafter, CET) employ the alternating-offer wage bargaining (AOB) mechanism within an estimated New Keynesian DSGE model for the US data and find that the model fits the key data moments well. However, they find that the same model but with Nash wage bargaining, though inferior, yields a close fit of the key data moments compared to the AOB specification. This result differs from the one obtained in the present paper for Latvia. Moreover, CET do not estimate the shock to matching efficiency. The different results obtained for the US and Latvia call for more studies across economies. Yet it is instructive to test the AOB model using the Latvian data.

1. INTRODUCTION

The standard business cycle approach of modelling labour markets without explicit unemployment (as in Erceg, Henderson and Levin (2000); hereinafter, EHL) has its drawbacks, of which the main one is that it has no implications for unemployment (so-called extensive margin of labour supply), and, thus, the variation of total hours worked is solely attributed to the variation in hours per employee (intensive margin). Second, it also tends to induce too little persistence in hours worked (see, e.g. Buss (2015)).

In reality, much of the variation in total hours worked is generated by the extensive margin of labour supply. To quantify it, this paper applies simple data variance decomposition to the Latvian data for the period of Q1 2002–Q4 2012 as detailed in the next Section. Though the data are noisy and, thus, the decomposition is rough, according to it, more than a half of the variation in total hours worked is explained by the variation in the number of employees. The two employment margins have different economic policy implications, and, thus, it is useful to distinguish between them in economic analysis.

The search-and-matching theory has become the most widespread economic theory of labour market since Merz (1995) and Andolfatto (1996) integrated the original Diamond–Mortensen–Pissarides framework into a standard general equilibrium model. The merit of search-and-matching models is due to the fact that market-clearing real business cycle models were unable to explain unemployment and the co-existence of unfilled vacancies and unemployed workers.

Nevertheless, the work of Shimer (2005) started a lively discussion of whether this theory can fit the data. Shimer (2005) concludes that the model in its basic form cannot fit the second moments of unemployment and vacancies. Many types of corrections to the model, such as sticky wage (Hall (2005a)), on-the-job search (Mortensen and Nagypál (2007)), high value of leisure (Hagedorn and Manovskii (2008)) and alternating-offer wage bargaining (Hall and Milgrom (2008)), have been proposed. Many of the proposals are united by the mechanism they affect the "fundamental surplus" – the fraction of firm profits allocated to create vacancies, which is the source of amplification and persistence of unemployment in these models (Ljungqvist and Sargent (2015)).

Much of the lengthy literature is devoted to either calibrated models or studies of the US data. Rarely the models are estimated using non-US data, and even more so using a full-fledged model. This paper adds to the literature by studying the ability of a richly specified New Keynesian DSGE model with search-and-matching frictions to fit the key moments of unemployment and vacancies, particularly for a non-US country. The model in this paper is closest to CTW (2011). The innovation of this paper compared to CTW is that it 1) adjusts the model to a member country of a currency union, 2) estimates the model for Latvia, and most importantly 3) studies the model's ability to fit both unemployment and vacancies simultaneously, not in isolation. This is done by using two specifications of the model: in one of them, the vacancies data are unobserved and the matching function is calibrated, resembling the CTW specification but with implications for (latent) vacancies; in the other specification, the vacancies data are observed and the matching function, including the shock to the matching efficiency, is estimated.

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The paper is structured as follows. Section 2 overviews the model. Section 3 describes the estimation procedure and the results. Section 4 concludes. Appendices contain more information about the model, its estimation and the results.

2. THE MODEL IN BRIEF

This paper adds the labour market frictions block of CTW (2011) to the model with financial frictions block of Buss (2015) which serves as a benchmark.

Since the model is almost a replica of CTW, this Section is a brief introduction to the model, whereas its formal description is relegated to Appendix C. The only noticeable difference between the CTW model and this one is in the behaviour of monetary authority which is modelled as a currency union in this paper.

2.1 Benchmark financial frictions model

The financial frictions model consists of the core block and the financial frictions add-in.

The core block builds on Christiano, Eichenbaum and Evans (2005) and Adolfson, Laséen, Lindé and Villani (2008). The three final goods – consumption, investment and exports – are produced by combining the domestic homogeneous good with specific imported inputs for each type of final good. Specialised domestic importers purchase a homogeneous foreign good, which they turn into a specialised input and

sell to domestic import retailers. There are three types of import retailers. One uses specialised import goods to create a homogeneous good used as an input into production of specialised exports. Another uses specialised import goods to create an input used in the production of investment goods. The third uses specialised imports to produce a homogeneous input used in the production of consumption goods. Exports involve a Dixit-Stiglitz (Dixit and Stiglitz (1977)) continuum of exporters, each of whom is a monopolist that produces a specialised export good. Each monopolist produces its export good using a homogeneous, domestically produced good and a homogeneous good derived from imports. The homogeneous domestic good is produced by a competitive, representative firm. The domestic good is allocated between government consumption (which consists entirely of the domestic good) and the production of consumption, investment and export goods. A part of the domestic good is lost due to the real friction in the model economy due to investment adjustment and capital utilisation costs. Households maximise the expected utility from a discounted stream of consumption (subject to habit) and leisure. In the core block, households own the economy's stock of physical capital. They determine the rate at which the capital stock is accumulated and the rate at which it is utilised. The households also own the stock of net foreign assets and determine its rate of accumulation.

Monetary policy is conducted as a hard peg of the domestic nominal interest rate to the foreign nominal interest rate. The government spending grows exogenously. Taxes in the model economy are: capital tax, payroll tax, consumption tax, labour income tax, and bond tax. Any difference between government spending and tax revenue is offset by lump-sum transfers.

The foreign economy is modelled as a structural vector autoregression (SVAR) in foreign output, inflation, nominal interest rate and technology growth. The model economy has two sources of exogenous growth: neutral technology growth and investment-specific technology growth.

The financial frictions add-in attaches to the above core block the financial frictions of Bernanke, Gertler and Gilchrist ((1999); hereinafter, BGG). Financial frictions suggest that borrowers and lenders are different people and that they have different information. Thus, the model introduces "entrepreneurs", i.e. agents who have a special skill in the operation and management of capital. Their skill in operating capital is such that it is optimal for them to operate more capital than their own resources can support by borrowing additional funds. There is some financial friction, because the management of capital is risky, i.e. entrepreneurs can go bankrupt, and only the entrepreneurs costlessly observe their own idiosyncratic productivity. In this block, households deposit money in banks. The interest rate that households receive is nominally non state-contingent.¹ Banks then lend funds to entrepreneurs using a standard nominal debt contract, which is optimal given the asymmetric information.² The amount that banks are willing to lend to an

¹ These nominal contracts give rise to wealth effects of unexpected changes in the price level, as emphasised by Fisher (1933). E.g., when a shock which drives the price level down occurs, households receive a wealth transfer. This transfer is taken from entrepreneurs whose net worth is thereby reduced. With tightening of their balance sheets, the ability of entrepreneurs to invest is reduced, and this generates an economic slowdown.

² Namely, the equilibrium debt contract maximises the expected entrepreneurial welfare, subject to zero profit condition on banks and specified return on household bank liabilities.

entrepreneur under the debt contract is a function of entrepreneurial net worth. This is how balance sheet constraints enter the model. When a shock that reduces the value of entrepreneurs' assets occurs, this cuts into their ability to borrow. As a result, entrepreneurs acquire less capital, and this translates into a reduction in investment and leads to a slowdown in the economy. Although individual entrepreneurs are risky, banks are not.

The financial frictions block brings in two new endogenous variables, of which one is related to the interest rate paid by entrepreneurs and the other - to their net worth. There are also two new shocks - one to idiosyncratic uncertainty and the other to entrepreneurial wealth.

2.2 Full model with labour market frictions block

We apply simple data variance decomposition to the Latvian data for the period of Q1 2002–Q4 2012.³

 $Var(H_t) = Var(\varsigma_t) + Var(L_t) + 2Covar(\varsigma_t, L_t)$

where H_t denotes total hours worked, ς_t stands for hours per employee, L_t is the number of people employed, Var is variance and Covar – covariance. H_t and L_t are in per capita terms, H_t and ρ_t are normalised by average hours worked, and all series are logged. Though the data are noisy and, thus, the decomposition is rough, according to it, about 58% of the variation in total hours is explained by the variation in employment, 28% is attributed to the variation in hours per employee, and about 14% – to the covariance term. Therefore, this paper adds the labour market search and matching framework of Mortensen and Pissarides (1994), Hall (2005a, 2005b) and Shimer (2005, 2012) with Taylor-type nominal wage rigidity as modelled by CTW to the benchmark financial frictions model of Buss (2015). A key feature of this model is that there are wage setting frictions, but they do not have a direct impact on on-going employer-employee relations as long as these are mutually beneficial⁴. However, wage setting frictions have an impact on the effort of an employer in recruiting new employees⁵. Accordingly, the setup is not vulnerable to Barro critique (1977) that wages cannot be allocational in on-going employeremployee relationships. Also, the intensive margin of labour supply is allowed, as is the endogenous separation of employees from their jobs.

As in the benchmark financial frictions model, there is the Dixit–Stiglitz specification of homogeneous goods production. A representative competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists who hire labour and capital services in competitive factor markets. The intermediate goods firms are assumed to be subject to the same Calvo price setting frictions as in the benchmark model. The search and matching framework dispenses with the specialised labour services abstraction and the accompanying monopoly power in the benchmark model. Labour

³ The sample period is constrained by data availability.

 ⁴ That is, the existence of the nominal wage frictions does not imply that the employer–employee relations are enforced upon them, since they can separate if their relationship is not beneficial.
 ⁵ The Nash wage depends on the relative bargaining power of the employer and the employee. The smaller

⁵ The Nash wage depends on the relative bargaining power of the employer and the employee. The smaller the relative bargaining power of the employee, the smaller the Nash wage is, and thus the incentive to recruit new employees is larger.

services are instead supplied by employment agencies – a modelling construct best viewed as a goods producing firm's human resource division, to the homogeneous labour market where they are bought by the intermediate goods producers.⁶

Each employment agency retains a large number of workers. Each employment agency is permanently allocated to one of N = 4 different equal-sized cohorts. Cohorts are differentiated according to the period (quarter) in which they renegotiate their wage. The nominal wage paid to an individual worker is determined by Nash bargaining, which occurs once every N periods.⁷ Since there is an equal number of agencies in each cohort, 1/N of the agencies bargain in each period. The intensity of labor effort is determined efficiently by equating the worker's marginal cost to the agency's marginal benefit. The assumption of efficient provision of labour on the intensive margin without any direct link to the sticky wage allows for a high frequency disconnect between wages and hours worked. Fundamentally, this model reflects that labour is not supplied on a spot market but within long-term relationships.

In an employment agency, the events during the period take place in the following order. At the beginning of the period, an exogenously determined fraction of workers is randomly selected to separate from the agency and go into unemployment. Also, a number of new workers arrive from unemployment in proportion to the number of vacancies posted by the agency in the previous period. Then, the economy's aggregate shocks materialise. After that, each agency's nominal wage rate is set. When a new wage is set, it evolves over the subsequent N-1periods. The wage negotiated in a given period covers all workers employed at an agency for each of the subsequent N - 1 periods, even those that will arrive later. Next, each worker draws an idiosyncratic productivity shock. A cut-off level of productivity is determined, and workers with lower productivity are laid off.⁸ After the endogenous layoff decision, the employment agency posts vacancies, and the intensive margin of labour supply is chosen efficiently by equating the marginal value of labour services to the employment agency with the marginal cost of providing them by the household. At this point, the employment agency supplies labour to the labour market.

The explicit description of the model is relegated to Appendix C.

⁶ The change leaves equilibrium conditions associated with the production of homogeneous good unaffected. Key labour market activities, e.g. vacancy postings, layoffs, labour bargaining, setting the intensity of labour effort, are all carried out inside employment agencies. Each household is composed of many workers, each of whom is in the labour force. A worker begins the period either unemployed or employed with a particular agency with a probability that is proportional to the efforts made by the agency to attract workers. Workers are separated from employment agencies either exogenously or because they are actively cut. Workers pass back and forth between unemployment and employment, but there are no agency to agency transitions.

⁷ The bargaining arrangement is atomistic, so that each worker bargains separately with a representative of the employment agency.

⁸ This is the endogenous part of separation as opposed to the exogenous separation mentioned at the beginning of the paragraph. From a technical point of view, this modelling is symmetric to the modelling of entrepreneurial idiosyncratic risk and bankruptcy. Two mechanisms by which the cut-off is determined are considered. One is based on the total surplus of a given worker, and the other is based purely on the employment agency's interest.

3. ESTIMATION AND RESULTS

The quarter is used as a time unit. A subset of model parameters is calibrated and the rest are estimated using the data for Latvia (domestic part) and the euro area (foreign part). To save space, calibration details are relegated to Appendix A.

The model is estimated with the Bayesian techniques. Two versions of the model will be discussed. In one version, the model is fed with 19 observables including the (quarterly growth rates of) unemployment rate, but vacancies data are not observed and the parameters in the matching function are calibrated. In the other version, the model is fed with 20 observables, including the data on both unemployment and vacancies. In the latter version, the parameters in matching function are estimated, including the shock to the matching efficiency. Prior-posterior information is relegated to Appendix A.

3.1 Unobserved vacancies

If vacancies are not observed, then the parameters of matching function are calibrated. The Cobb–Douglas matching function is of the following form:

$$m_t = \sigma_m (1 - L_t)^\sigma v_t^{1 - \sigma} \tag{1}$$

where m_t denotes the total matches, L_t – the fraction of employed, v_t – total vacancies, σ_m – level parameter, σ – unemployment share. The particular calibration results in $\sigma_m = 0.4$ and $\sigma = 0.5$ (see Table 1). This calibration together with the rest of model parameter values reported in Appendix A yields the following results.

Description	Vacancies unobserv- able	Vacancies observable							
	Calibrated		Prior		Poste	erior	HPD in	nterval	
		Distr.	Mean	St. d.	Mean	St. d.	10%	90%	
Unemployment share σ	0.500	β	0.5	0.05	0.373	0.017	0.326	0.417	
Level parameter σ_m	0.400	β	0.4	0.05	0.394	0.024	0.332	0.442	
Shock standard deviations									
Matching efficiency	0	Inv-Γ	0.1	inf	12.810	1.429	10.925	14.624	

Table 1Parameters of matching function

Model and data moments

The model-implied standard deviation of the first differenced unemployment rate is 10.35 versus 9.75 in the sample data (see Table 2). The second-moment fit is closer than that for the US reported by Shimer (2005). This is determined by at least two sources: 1) the assumed wage stickiness (as emphasised by Hall (2005a)) a la Taylor, and 2) the high estimated replacement ratio (0.80 at the posterior mean), as emphasised by Hagedorn and Manovskii (2008).

The model-implied correlation between the first differences of unemployment and the vacancies is also decent (-0.40) though lower than in the sample data (-0.54).

Table 2Data and model moments

	corr(Δu,Δv)	St. d. Δu	St. d. Δv
Data	-0.54 [-0.68 -0.35]	9.75 [8.37 11.68]	16.04 [13.76 19.21]
Model without vacancies	-0.40 [-0.42 -0.37]	10.35 [10.15 10.55]	46.62 [45.72 47.55]
Model with vacancies	-0.30 [-0.32 -0.27]	9.48 [9.29 9.67]	36.19 [35.50 36.91]

Notes: Statistics for data are calculated using 71 observations long samples, and those for the two models are calculated using 5 000 observations long simulated data at the posterior mean. 95% confidence interval is given in brackets.

However, the above two moments are fitted at the cost of too volatile vacancies: the model-implied standard deviation of the first differenced vacancies is 2.9 times larger than in the sample data (46.6 vs. 16.0).⁹

Conditional variance decomposition

The conditional variance decomposition indicates that 3/4 of the variance of first differenced unemployment rate at an eight-quarter forecast horizon are explained by the markup shock to imports for exports (35.9%), the markup shock to imports for investment (18.1%), the labour preference shock (13.3%), and the stationary technology shock (5.4%), while 4/5 of hours per employee are explained by the labour preference shock alone (see Table 3, last two columns).

Table 3

Conditional var	iance decompos	ition given mo	del paramete	r uncertainty	at eight-qu	larter for	recast
horizon (%; post	erior mean)						

	Description	Model	R	π^{c}	GDP	С	Ι	NX GDP	Н	w	q	Ν	Spread	$\frac{H}{L}$	U
ε_t	Stationary	finfric	0.0	0.6	0.7	0.1	0.0	0.5	8.6	0.3	0.6	0.1	0.1		
	technology	full	0.1	3.0	8.9	0.5	0.1	2.5	5.9	1.9	2.7	0.5	0.4	1.2	5.4
Υ_t	MEI	finfric	0.1	0.0	1.8	0.1	19.2	3.4	3.2	0.2	0.0	12.7	12.2		
		full	0.1	0.3	2.6	0.2	38.4	6.4	2.1	1.5	0.3	12.3	13.8	0.2	2.7
ζ_t^c	Consumption	finfric	0.2	0.0	7.1	78.7	0.1	14.8	5.6	0.0	0.0	0.1	0.1		
	preferences	full	0.7	0.8	3.9	83.3	0.1	23.5	4.7	1.6	0.7	0.3	0.2	6.6	3.2
ζ_t^h	Labour	finfric	0.1	10.1	5.8	4.0	1.0	6.6	8.0	51.7	9.3	2.0	0.6		
	preferences	full	0.1	3.2	7.9	1.9	0.1	2.4	19.8	12.2	2.9	0.6	0.3	84.0	13.3
g_t	Government	finfric	0.0	0.0	8.2	0.0	0.0	0.1	6.6	0.0	0.0	0.0	0.0		
	spending	full	0.0	0.2	2.8	0.0	0.0	0.5	2.0	0.9	0.2	0.0	0.1	0.3	1.7
$ au_t^d$	Markup,	finfric	0.0	22.7	2.2	0.1	0.1	0.3	1.8	33.0	20.8	0.5	0.1		
	domestic	full	0.0	23.6	1.2	0.1	0.1	0.1	1.1	46.6	21.6	0.8	0.1	0.1	2.5
τ_t^x	Markup, exports	finfric	0.0	0.0	2.3	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.0		
		full	0.0	0.4	1.9	0.0	0.1	0.0	1.6	1.5	0.4	0.2	0.1	0.2	2.6
$ au_t^{mc}$	Markup, imports	finfric	0.0	59.1	4.1	0.1	0.0	1.1	3.2	3.9	54.1	0.1	0.0		
	for consumption	full	0.0	53.8	0.9	0.1	0.0	0.6	0.8	2.3	49.1	0.1	0.0	0.1	1.5
$ au_t^{mi}$	Markup, imports	finfric	0.1	0.3	23.3	0.0	5.4	6.0	34.4	0.1	0.3	7.9	6.1		
	for investment	full	0.1	4.2	15.2	0.0	8.7	7.0	26.4	3.6	3.8	16.5	14.4	2.4	18.1
τ_t^{mx}	Markup, imports	finfric	0.1	0.0	28.4	0.1	0.1	6.2	22.8	0.1	0.0	0.2	0.1		
	for exports	full	0.3	3.5	40.3	0.1	0.3	9.0	29.4	16.3	3.2	1.0	0.8	2.6	35.9

⁹ The volatility of vacancies is not reduced substantially when the share of cost of vacancy creation in the total cost of meeting a worker is raised from zero to 20%.

	Description	Model	R	π^{c}	GDP	С	Ι	MX GDP	Н	w	q	Ν	Spread	$\frac{H}{L}$	U
γ_t	Entrepreneurial	finfric	0.7	0.6	10.1	0.2	58.1	38.9	1.1	0.6	0.5	62.4	77.3		
	wealth	full	0.9	0.3	7.9	0.1	36.2	31.7	0.7	1.1	0.2	50.4	66.0	0.3	1.0
$ ilde{\phi}_t$	Country risk	finfric	91.8	0.4	2.2	5.5	8.7	17.8	0.8	3.1	0.4	9.4	1.7		
	premium	full	91.0	0.4	1.3	1.6	7.5	12.7	0.9	3.1	0.3	11.9	1.7	1.5	3.7
$\mu_{z,t}$	Unit-root	finfric	1.7	0.0	0.2	0.0	0.1	1.4	0.0	0.2	0.2	0.1	0.0		
	technology	full	1.6	0.0	0.3	0.0	0.1	1.3	0.0	0.5	0.2	0.1	0.0	0.1	0.0
$\mathcal{E}_{R^*,t}$	Foreign interest	finfric	1.6	0.1	0.1	0.3	0.3	0.7	0.0	0.1	0.0	0.2	0.0		
	rate	full	1.6	0.1	0.0	0.1	0.2	0.6	0.0	0.1	0.0	0.3	0.0	0.1	0.1
$\varepsilon_{y^*,t}$	Foreign output	finfric	3.6	0.1	0.0	0.8	0.2	2.1	0.0	0.3	0.2	0.0	0.0		
		full	3.5	0.2	0.1	0.4	0.2	1.6	0.1	0.1	0.1	0.0	0.0	0.3	0.1
$\mathcal{E}_{\pi^*,t}$	Foreign inflation	finfric	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0		
		full	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.0
	5 foreign*	finfric	98.7	0.6	2.6	6.6	9.3	22.0	0.8	3.6	0.9	9.7	1.8		
		full	97.7	0.7	1.8	2.2	8.0	16.3	1.0	3.9	0.7	12.3	1.8	1.9	4.0
	All foreign**	finfric	98.9	60.1	60.7	6.8	14.9	35.3	63.1	7.8	55.3	17.9	8.0		
		full	98.1	62.6	60.1	2.4	17.0	32.8	59.2	27.5	57.3	30.2	17.1	7.3	62.1
	Measurement	finfric	0.0	5.8	3.4	10.0	6.6	0.0	1.9	6.4	13.5	4.2	1.7		
	error	full	0.0	6.0	4.7	11.7	7.9	0.0	4.4	6.8	14.1	5.0	2.1	0.0	8.1

Table 3 (cont.)

Notes: R – nominal interest rate, π^c – CPI, C – real private consumption, I – real investment, $\frac{NX}{GDP}$ – net exports to GDP ratio, H – total hours worked, w – real wage, q – real exchange rate, N – net worth, Spread – interest rate spread, $\frac{H}{L}$ – hours per employee, U – (first differenced) unemployment rate. "finfric" denotes benchmark financial frictions model, "full" stands for full model with unemployment.

* "5 foreign" is the sum of the foreign stationary shocks, R_t^*, π_t^*, Y_t^* , the country risk premium shock, $\tilde{\phi}_t$, and the world-wide unit root neutral technology shock, $\mu_{z,t}$.

** "All foreign" includes the above five shocks as well as the markup shocks to imports and exports, i.e. τ_t^{mc} , τ_t^{mi} , τ_t^{mi} , τ_t^{mx} and τ_t^x .

Unemployment in impulse response analysis

One of the main benefits of having unemployment in a general equilibrium model is to be able to study the effects of various shock scenarios on unemployment. The analysis below serves as an illustrative example of such an analysis.

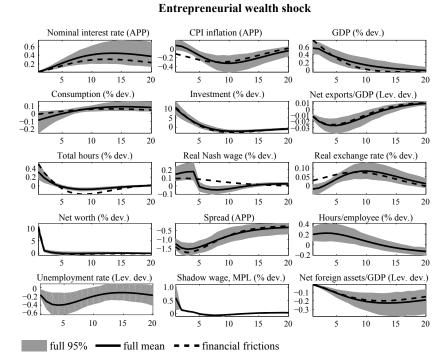
Since Table 3 shows that the entrepreneurial wealth shock¹⁰ is one of the key drivers of the variance of investment, it is instructive to discuss the impulse response functions (IRFs) of this shock. The IRFs to entrepreneurial wealth shock are plotted in Figure 1, which shows that a positive temporary entrepreneurial wealth shock drives up net worth, reduces interest rate spread, and, thus, increases investment (by about the same percentage change as in net worth); GDP goes up accordingly, and so do the real wage and total hours worked. Both exports and imports increase, but the latter increases more due to the demand for investment goods, and, thus, the net exports to GDP ratio decreases slightly. As a consequence, the net foreign assets to GDP ratio decreases, driving slightly up the risk premium on the domestic nominal interest rate. Inflation goes down, and the real exchange rate depreciates.

¹⁰ For example, a shock to entrepreneur's asset price.

The above results are broadly similar across the two models, but the addition of the labour block allows us to study the effects on the labour market: unemployment rate drops, and hours per employee increase.¹¹

Figure 1

Impulse responses to entrepreneurial wealth shock



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Historical shock decomposition

Figure 2 shows the decomposition of unemployment rate. The model predicts that the main driving forces of unemployment during the 2005 boom were the labour and the consumption preference shocks, while during the 2008 recession these same shocks together with the country risk premium and the markup shocks to imports for exports drove unemployment upwards.

¹¹ Here and in other IRFs of the full model, the real wage rate jumps after around four quarters, and this is the artefact of the Taylor-type modelling of nominal wage rigidity. In particular, wages are renegotiated every four quarters, in a staggered way. Therefore, after a shock has occurred, some of the employment agencies are stuck with wages that they set before the shock hit. Depending on how much a wage adjustment is needed, it can be quite vigorous when the "second to last" or "last" employment agency have their turns to set their wages optimally. Such dynamics of the modelled real wage can be considered as implausible and suggests that the Taylor-type frictions may be too strict for the particular sample of Latvian data. Whereas the Taylor-type frictions might be a reasonable approximation of reality in normal times, it appears to fail during the great recession episode when the real (and nominal) wage was rather flexible in Latvia. This evidence calls for revision of the way wage rigidity is modelled.

Meanwhile, the IRF figure contains also the shadow wage, or the marginal product of labour (MPL), i.e. the real wage the entrepreneurs would be willing to pay their workers absent the wage rigidity. In this and other IRFs, the shadow wage reacts more sharply than wages in both the benchmark and the full models. Also, the shadow wage adjusts more rapidly, i.e. its dynamics often dies out within a year, while the rigid wages continue to adjust.

The role of the markup shock to imports for exports might require explanation. During 2006–2008, this shock was persistently positive, raising pressure for the substitution of imported inputs for domestic inputs in the production of exports, and, thus, lowering unemployment. However, during the period of 2009–2012, this shock is estimated to be persistently negative. Such a development in relative prices of inputs boosted imports for production of exports; consequently, foreign trade grew substantially but partly at the expense of a slower growth of the domestic production, thus contributing to higher unemployment.¹²

In comparison with the results of CTW for Sweden, there are differences in driving forces of unemployment between the two countries. For Sweden, the entrepreneurial wealth, exports markup and consumption preference shocks drew unemployment down during the pre-recession period of 2007–2008, while these same shocks contributed much to the reverse process during the period of great recession.

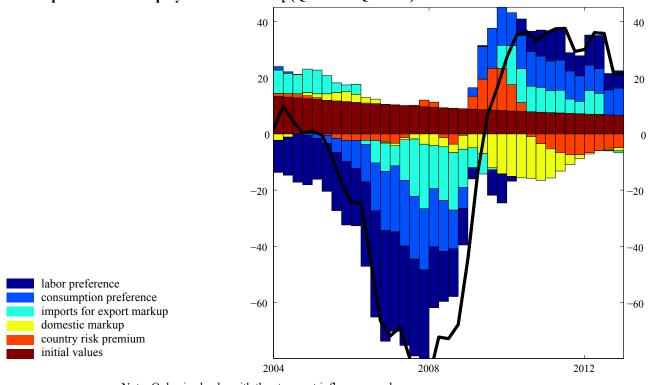


Figure 2 **Decomposition of unemployment rate 1** – L_t (Q1 2004– Q4 2012)

Note: Only six shocks with the strongest influence are shown.

Forecasting performance

Figure 3 shows one-quarter ahead forecasts of the full and the benchmark financial frictions models for selected observables.¹³

¹² Having said that, the contribution of the markup shock to imports for exports diminishes, if variances of data measurement errors are estimated rather than calibrated to explain 10% of data variances (the results are not reported due to space constraints); hence it is not clear how much of this shock represents a structural shock and how much – a model misspecification or data measurement error.

¹³ These are not true out-of-sample forecasts because the models are calibrated/estimated on the whole sample period of Q1 1995–Q4 2012. Nevertheless, these figures indicate approximate forecasting performance of the models. Particularly, it is informative to see whether the models tend to yield unbiased forecasts and how the addition of labour block affects them.

The introduction of labour block apparently improves the one-step ahead forecasts for total hours worked and, thus, also for GDP; volatility of these both variables has been reduced. The forecasts of quarterly growth rate of unemployment are decent, and those of (latent) quarterly growth rate of vacancies, though highly volatile, have reasonable business cycle dynamics.

Figure 3 One-step ahead forecasts (selected)

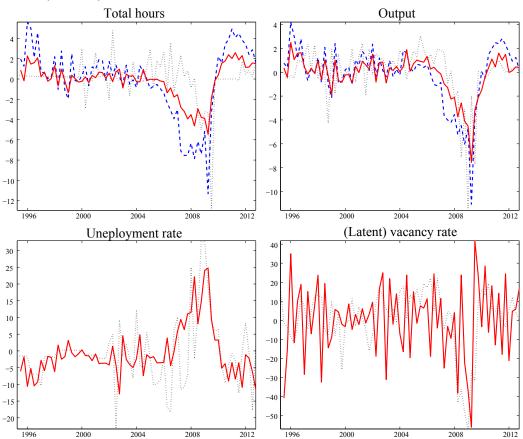




Table 4 reports the forecasting performance of full and benchmark models relative to a random walk model (in terms of quarterly growth rates) with respect to predicting CPI inflation and GDP for horizons of one, four, eight and 12 quarters. Table 4 also reports the forecasting performance of a Bayesian SVAR (with the same structure as the foreign SVAR, and with similar priors), since it is often taken as a benchmark in the literature¹⁴.

Table 4 shows that the model forecasts both variables at least as precisely as the random walk model does at all the horizons considered, and its relative performance improves with a higher horizon. Moreover, the full model tends to outperform the benchmark financial frictions model at a one quarter horizon in GDP forecasting, likely due to the more persistent modelled total hours worked. The performance of the full model is roughly comparable to that of the Bayesian SVAR.

¹⁴ The particular SVAR has some economically implausible estimated parameters, since GDP, CPI inflation and nominal interest rate data of Latvia do not possess a stable and economically plausible relationship over the particular sample span.

Table 4Forecasting performance

Model	Distance	1	1 Q		Q	8	Q	12 Q	
	measure	π^{c}	Δy						
Finfric	RMSE	1.00	0.96	0.78	0.70	0.65	0.64	0.68	0.64
	MAE	0.93	1.15	0.80	0.69	0.70	0.57	0.66	0.60
Full	RMSE	0.93	0.75	0.85	0.71	0.66	0.64	0.69	0.64
	DM p-val	0.080	0.003	0.891	0.680	0.666	0.628	0.602	0.647
	MAE	0.89	0.85	0.89	0.67	0.70	0.57	0.67	0.58
	DM p-val	0.152	0.000	0.944	0.184	0.510	0.422	0.565	0.172
SVAR	RMSE	0.86	0.72	0.71	0.81	0.62	0.68	0.63	0.66
	MAE	0.89	0.71	0.70	0.77	0.62	0.62	0.58	0.61

Notes: For RMSE and MAE, a number less than unity indicates that the model produces more precise forecasts compared to the random walk model. DM p-val is a one-sided p-value of the Diebold–Mariano (Diebold and Mariano (1995)) test for equal forecast accuracy between the full and the financial frictions models. The value below 0.05 implies that the precision of model's forecast is better than that of the alternative model at a 5% significance level. The results show that the full model's one-quarter ahead forecasts of GDP are statistically significantly more precise than those of the benchmark financial frictions model. SVAR is estimated on three domestic variables: GDP, CPI and nominal interest rate, and is of the same structure and with similar priors as the foreign SVAR. Note that this is not a true out-of-sample forecasting performance, since the models have been estimated on the whole sample period of Q1 1995–Q4 2012. "finfric" – benchmark financial frictions model, "full" – full model with unemployment, "SVAR" – Bayesian SVAR model serving as another benchmark.

Latent labour market variables

This subsection ends with a few smoothed latent labour market variables, shown in Figure 4. The smoothed probability of filling a vacancy within a quarter (see Figure 4, upper left panel) overshoots on a few occasions but otherwise looks reasonable.

The Cobb–Douglas labour matching technology employed by CTW is a popular choice in the literature, but it does not ensure that the matching probability is proper, i.e. bounded in the interval [0,1]. Therefore, den Haan, Ramey and Watson ((2000); hereinafter, dHRW) came up with an alternative matching technology which ensures a proper matching probability. For robustness, Figure 4 shows the results for both matching functions, with dHRW matching function being:

$$m_t = \frac{(1-L_t)v_t}{((1-L_t)^l + v_t^l)^{\frac{1}{l}}}$$
(2),

with a particular calibrated value l = 1.36 for the Latvian data¹⁵. If not clearly stated otherwise, all the other results are produced using the Cobb–Douglas specification.

The steady-state value of quarterly job finding rate is 0.28, but its smoothed value (see Figure 4, upper right panel) overshoots significantly during the boom period right before the 2008 recession. This is because the smoothed level of unemployment rate¹⁶ is underestimated during that period (see Figure 4, bottom panel).

¹⁵ dHRW use l = 1.27 for the US data.

¹⁶ The model is fed with quarterly growth rates of unemployment.

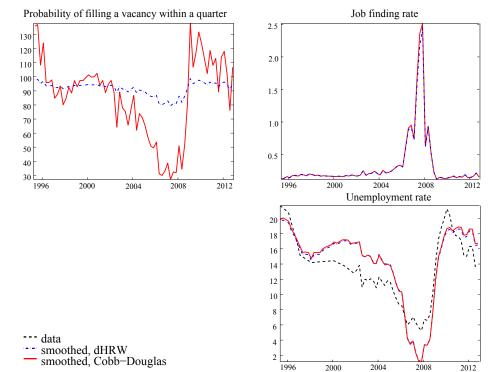


Figure 4 Latent labour market variables, Cobb–Douglas versus dHRW matching function

Notes: Probability of filling a vacancy within a quarter is defined as the ratio of total job matches over total job vacancies times 100, $Q = 100 \frac{m_t}{v_t}$. Job finding rate within a quarter is defined as the ratio of total job matches over total unemployment $f_t = \frac{m_t}{1-L_t}$. Unemployment rate is "latent", as the model is fed with the first differences of unemployment rate.

3.2 Observed vacancies

This subsection adds the quarterly growth rate of vacancies as an observable and estimates the matching function together with an AR(1) shock to matching efficiency. It turns out that this brings a few results deemed implausible and, thus, highlights a possible model misspecification.

First, the posterior mean of unemployment share in the matching function decreases to 0.37 (from a prior 0.5; see Table 1), i.e. outside the range considered to be sound [0.5, 0.7] (Shimer (2005)).

Second, during the boom period of 2004–2007, one-period-ahead forecasts of vacancies display dynamics opposite to the data, i.e. the forecasted vacancies tend to decrease, generating positive correlation between vacancies and unemployment, whereas the data increase during this period (see Figure 5, left panel). This result is due to the attempt of the model to fit the volatility of vacancies, which now is slightly closer to the data yet still 2.2 times higher (see Table 2). However, the better fit of vacancy volatility comes at the cost of a worse fit of the correlation between (first differenced) unemployment and vacancies, which decreases from -0.40 to -0.30 (see Table 2). The one-quarter-ahead forecasts of vacancies are to be compared to those in the previous subsection where they, though having too high short-term volatility, clash less with the data in business cycle frequencies (see Figure 3, bottom right panel).

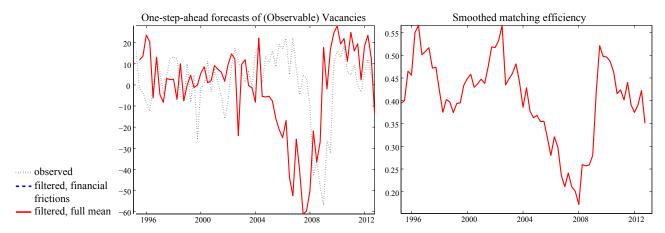


Figure 5 Implausible results in presence of shock to matching efficiency

Third, the smoothed AR(1) process of matching efficiency is counter-intuitively counter-cyclical: the matching efficiency, which commonly is referred to be negatively related to structural unemployment, decreases during the 2004–2007 boom, and increases thereafter during recession (see Figure 5, right panel).

4. SUMMARY AND CONCLUSIONS

This paper examines, in an estimated, full-fledged New Keynesian DSGE model with Nash wage bargaining, sticky wage and high value of leisure akin to CTW (2011), whether search-and-matching frictions in the labour market can explain the aggregate labour market dynamics in Latvia. The paper adds to the literature by studying the ability of a richly specified New Keynesian DSGE model with search-and-matching frictions to fit the key moments of unemployment and vacancies, particularly for a non-US country.

The results are as follows. If vacancies are not observed, the model can, to a reasonable degree, generate realistic variance and dynamics of unemployment, and correlation between unemployment and (latent) vacancies but at the expense of too volatile vacancies. As a by-product, one-quarter-ahead forecasts of hours worked and GDP exhibit less excess volatility and, thus, are more precise, compared to a model without search-and-matching frictions.

However, if both unemployment and vacancies are observed and a shock to the matching efficiency is allowed for, then the cyclical behaviour of forecasted vacancies as well as correlation between unemployment and vacancies tend to counter the data (for the benefit of a better fit of vacancy volatility), and the smoothed matching efficiency is counter-intuitively counter-cyclical.

The results tend to be different than for the US (e.g. by CET), calling for more studies across economies. As the next step, it is instructive to test the AOB model for the Latvian data.

APPENDICES

Appendix A CALIBRATION AND ESTIMATION DETAILS

For space considerations, the information regarding the first specification (with calibrated matching function) is shown. The results for the model without search-and-matching frictions are taken from Buss (2015).

A1. Calibration

The calibrated values are displayed in Tables A1 and A2. These are the parameters that are typically calibrated in the literature and are related to "great ratios" and other observable quantities related to steady state values. The values of parameters are selected such that they would be specific to the data at hand. Sample averages are used when available. We are using the calibrated values of Buss (2015) for the parameters common for full and benchmark financial frictions models.

Table A1

Calibrated parameters

Parameter	Value	Description
Core block		
α	0.400	Capital share in production
β		Discount factor
ω _c	0.450	Import share in consumption goods
ω_i		Import share in investment goods
ω_x		Import share in export goods
$ ilde{\phi}_a$	0.010	Elasticity of country risk to net asset position
η_g	0.202	Government spending share of GDP
τ_k	0.100	Capital tax rate
$ au_w$	0.330	Payroll tax rate
$ au_c$	0.180	Consumption tax rate
$ au_y$	0.300	Labour income tax rate
τ_b	0.000	Bond tax rate
μ_z	1.005	Steady state growth rate of neutral technology
μ_{ψ}	1	Steady state growth rate of investment technology
$\overline{\pi}$	1.005	Steady state gross inflation target
λ_i	1.300	Price markup for the domestic good and imports, i=d; m, c; m, i
$rac{\lambda_i}{\lambda_j}$	1.200	Price markups for exports, $j = x; m, x$
ϑ_w	1.000	Wage indexation to real growth trend
ù ^j	$1 - \kappa^j$	Indexation to inflation target for $j = d; x; m, c; m, i; m, x; w$
π		Third indexing base
$ ilde{\phi}_{\scriptscriptstyle S}$	0	Country risk adjustment coefficient
Financial fri	ctions bloc	k
$F(\overline{\omega})$	0.020	Steady state bankruptcy rate
$100W_e/y$		Transfers to entrepreneurs
Labour mark	et friction	s block
L	0.863	Steady state fraction of employment (1 – unemployment rate)
Ν	4	Number of agency cohorts/length of wage contracts
arphi		Curvature of hiring costs
ρ		Exogenous survival rate of a match
σ		Unemployment share in matching technology
σ_m		Level parameter in matching function
ι	1.000	Employment adjustment cost dependence on tightness V/U

The discount factor β and the tax rate on bonds τ_b are set to match roughly the sample average real interest rate for the euro area. The capital share α is set to 0.4. Import shares are set to reasonable values after consulting input-output tables and fellow economists and stand at 45%, 65% and 30% for consumption, investment and exports respectively.¹ The government spending share in GDP is set to match the sample average, i.e. 20.2%. The steady state growth rates of neutral technology and inflation are set to 2% annually and correspond to the euro area. The steady state growth rate of investment specific technology is set to zero. The steady state quarterly bankruptcy rate is calibrated to 2%, up from 1% in the CTW model for Swedish data. The values of price markups are set to the typical values found in the literature, i.e. to 1.2 for exports and imports for exports, and 1.3 for the domestic, imports for consumption and imports for investment. There is full indexation of wages to the steady state real growth $\vartheta_w = 1$. The other indexation parameters are set to get full indexation and thereby avoid steady state price and wage dispersion, following CTW. Tax rates are calibrated such that they would represent implicit or effective rates. Three of them are calibrated using Eurostat data²: the tax rate on capital income is set to 0.1, the value-added tax on consumption τ^{c} and the personal income tax rate that applies to labour τ^{y} are set to $\tau^{c} = 0.18$, and $\tau^{y} = 0.3$. The payroll tax rate is set to $\tau^w = 0.33$, down from the official 0.35 (0.24 of employer and 0.11 of employee). The elasticity of country risk to net asset position $\tilde{\phi}_a$ is set to a small positive number, and in that region its purpose is to induce a unique steady state for the net foreign asset position. Transfers to entrepreneur parameter W_e/y is kept the same as in CTW. The country risk adjustment coefficient in the uncovered interest parity (UIP) condition is set to zero in order to impose the nominal interest rate peg.

For the labour block, the steady state unemployment rate is set to the sample average. The length of wage contract N is set to annual negotiation frequency, as in CTW. The curvature of hiring costs is set to quadratic. The exogenous survival rate of the match is set to 0.97, similar to that in CTW, and to yield a reasonable steady state job finding rate of 0.28. The matching function parameter σ is set so that the number of unemployed and the number of vacancies both have equal factor shares in the production of matches³. The level parameter in matching σ_m is calibrated to 0.4, down from 0.57 in CTW, reflecting the fact that the natural level of unemployment in Latvia is higher than in Sweden. Its particular value is preferred by the model fit in terms of MDD. As in CTW, we assume the hiring costs, not the search costs, thus t = 1. Endogenous breakups are determined using employer surplus only.⁴

¹ The import share in exports has been reduced to 30% (from 55% in Buss (2015)) due to Stehrer (2013) who suggest, from the value-added perspective, that the share is about 30%. Such a change reduces somewhat the log marginal data density (by about one point) and the importance of the markup on the shock to imports for exports.

² Source: http://epp.eurostat.ec.europa.eu/cache/ITY_PUBLIC/2-29042013-CP/EN/2-29042013-CP-EN.PDF, cited 6 September, 2013.

³ Shimer (2005) estimates σ to be 0.72 for the US data. The so called Hosios condition relates this parameter one-to-one to the worker's bargaining power (see, e.g. Amaral and Tasci (2012)). ⁴ The above is bargained by the set of the

⁴ The choice is backed by a better model fit to the data. It is also the choice of CTW who argue that including worker surplus in the separation criteria would introduce a tendency for job separations to decrease at the beginning of recessions, as the value to the worker of holding on his/her job then increases, but this tendency appears to be counterfactual.

Three observable ratios are chosen to be exactly matched throughout the estimation, and therefore three corresponding parameters are recalibrated for each parameter draw: the steady state real exchange rate $\tilde{\varphi}$ to match the export share of GDP in the data, the scaling parameter for disutility of labour A_L to fix the fraction of time that individuals spend working⁵, and the entrepreneurial survival rate γ is set to match the net worth to assets ratio⁶. Comparing across the models, the implied posterior mean of the scaling parameter for disutility of labour is considerably higher for the model with unemployment than for the benchmark model.

In the earlier steps of calibration, the depreciation rate of capital δ was also set to match the ratio of investment over output, but the realised value of depreciation rate turned out to be rather high (unless the capital share in production α was substantially increased but that yielded an excessively high capital to output ratio) and sensitive to the initial values, therefore it was decided to fix the quarterly depreciation rate to a more reasonable value of 3%.

Table A2Matched moments and corresponding parameters

	Parameter description	Posterio	r mean	Moment	Moment
		finfric	full		value
\widetilde{arphi}	Real exchange rate	2.04	0.87	$SP^{x}X/(PY)$	0.462
A_L	Scaling of disutility of work	37.81	348524.09	Lς	0.240
γ	Entrepreneurial survival rate	0.96	0.96	$n/(p_k,k)$	0.600

Notes: Quarterly depreciation rate of capital is fixed at 3%. "finfric" denotes benchmark financial frictions model, "full" stands for full model with unemployment.

A2. Shocks and measurement errors

In total, there are 21 exogenous stochastic variables in the full model: four technology shocks – stationary neutral technology ε , stationary marginal efficiency of investment Y, unit-root neutral technology μ_z and unit-root investment specific technology μ_{Ψ} –, shock to consumption preferences ζ^c and to disutility of labor supply ζ^h , shock to government spending g, and country risk premium shock that affects the relative riskiness of foreign assets compared to domestic assets $\tilde{\phi}$. There are five markup shocks, one for each type of intermediate good τ^d , τ^x , $\tau^{m,c}$, $\tau^{m,i}$, $\tau^{m,x}$ (d – domestic, x – exports, m, c – imports for consumption, m, i – imports for investment, m, x – imports for exports). The financial frictions block has two shocks – one to idiosyncratic uncertainty σ , and one to entrepreneurial wealth γ . There are also shocks to each of the foreign observed variables – foreign GDP y^* , foreign inflation π^* , and foreign nominal interest rate R^* .

⁵ This calibrated fraction of time spent working differs between the benchmark and the full models: whereas it is 0.27 for the benchmark model, it is lowered to 0.24 for the full model due to the existence of unemployment in the latter. Both values are somewhat arbitrary but checked against the model fit with respect to their neighbouring values. ⁶ The net worth to assets ratio for Latvia, if the definition of CTW is taken, yields about 0.15. However, the

⁶ The net worth to assets ratio for Latvia, if the definition of CTW is taken, yields about 0.15. However, the model fit favours a much larger number, 0.6, which is used in the final calibration. The latter number might be rationalised, if the net worth is measured not only by the share price index but if it includes also the real estate value.

The employment frictions block adds three shocks: shock to the bargaining power of workers η , shock to the matching productivity σ_m , and shock to the productivity dispersion among workers affecting the endogenous job separations σ_a .

The stochastic structure of exogenous variables is the following.

11 of them evolve according to AR(1) processes:

 $\varepsilon_t, \Upsilon_t, \zeta_t^c, \zeta_t^h, g_t, \tilde{\phi}_t, \sigma_t, \gamma_t, \eta_t, \sigma_{m,t}, \sigma_{a,t}.$

Five shock processes are i.i.d.:

 $\tau^d_t, \tau^x_t, \tau^{m,c}_t, \tau^{m,i}_t, \tau^{m,x}_t,$

and five shock processes are assumed to follow a SVAR(1):

 $y_t^*, \pi_t^*, R_t^*, \mu_{z,t}, \mu_{\Psi,t}.$

Four shocks are suspended in the estimation, and they are shock to the unit-root investment specific technology $\mu_{\Psi,t}$, the idiosyncratic entrepreneurial risk shock σ_t , shock to the bargaining power η_t , and shock to the standard deviation of idiosyncratic productivity of workers $\sigma_{a,t}$. The first one should correspond to the foreign block, but its identification is dubious in the particular SVAR model. The second has been found to have limited importance in CTW. Also, CTW argue that shocks to η_t seem superfluous, as we already have the standard labour supply shock, i.e. labour preference shock ζ_t^h . In the model version where vacancies are not observed, the shock to matching technology $\sigma_{m,t}$ is also suspended.

There are measurement errors, except for the domestic interest rate and foreign variables. The variance of measurement errors is calibrated to correspond to 10% of the variance of each data series.

A3. Priors

There are 24 structural parameters, eight AR(1) coefficients, 16 SVAR parameters for the foreign economy, and 16 shock standard deviations estimated with the Bayesian techniques within Matlab/Dynare environment (Adjemian et al. (2011)). The priors are displayed in Tables A3–A6. The priors common to the benchmark financial frictions model are taken from Buss (2015). Regarding the three new parameters in the labour block, for hiring costs as a fraction of GDP *hshare* we use a prior with the mean of 0.3%, up from 0.1% in CTW, in order to move it closer to the posterior. The prior mean of *bshare*, the ratio of the flow value of utility provided to the household of a worker of being unemployed to the flow value of utility of a worker being employed, is 0.75, as in CTW. The prior mean of the endogenous employer–employee match separation rate *F*,% is 0.25%, i.e. roughly 7.7% of the total job separation rate, similar to CTW.

Table A3
Estimated foreign SVAR parameters

	Parameter description		Prior		Poste	erior	HPD i	nterval
		Distribu- tion	Mean	St. d.	Mean	St. d.	10%	90%
$ ho_{\mu_z}$	Persistence, unit root technology	β	0.50	0.075	0.590	0.063	0.487	0.696
<i>a</i> ₁₁	Foreign SVAR parameter	Ν	0.90	0.05	0.913	0.034	0.852	0.977
<i>a</i> ₂₂	Foreign SVAR parameter	Ν	0.50	0.05	0.521	0.055	0.438	0.605
<i>a</i> ₃₃	Foreign SVAR parameter	N	0.90	0.05	0.954	0.023	0.919	0.989
<i>a</i> ₁₂	Foreign SVAR parameter	N	-0.10	0.10	-0.165	0.091	-0.314	-0.016
<i>a</i> ₁₃	Foreign SVAR parameter	N	-0.10	0.10	-0.045	0.054	-0.124	0.037
<i>a</i> ₂₁	Foreign SVAR parameter	N	0.10	0.10	0.181	0.043	0.097	0.260
<i>a</i> ₂₃	Foreign SVAR parameter	N	-0.10	0.10	-0.090	0.055	-0.183	-0.008
<i>a</i> ₂₄	Foreign SVAR parameter	N	0.05	0.10	0.078	0.041	0.009	0.146
<i>a</i> ₃₁	Foreign SVAR parameter	Ν	0.05	0.10	0.080	0.029	0.032	0.131
<i>a</i> ₃₂	Foreign SVAR parameter	Ν	-0.10	0.10	-0.095	0.058	-0.198	0.002
<i>a</i> ₃₄	Foreign SVAR parameter	Ν	0.10	0.10	0.108	0.026	0.068	0.149
<i>C</i> ₂₁	Foreign SVAR parameter	Ν	0.05	0.05	0.021	0.040	-0.048	0.088
<i>c</i> ₃₁	Foreign SVAR parameter	Ν	0.10	0.05	0.145	0.031	0.094	0.196
<i>C</i> ₃₂	Foreign SVAR parameter	N	0.40	0.05	0.374	0.053	0.286	0.459
<i>C</i> ₂₄	Foreign SVAR parameter	N	0.05	0.05	0.065	0.046	-0.003	0.135
<i>c</i> ₃₄	Foreign SVAR parameter	N	0.05	0.05	0.048	0.034	-0.002	0.101

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

Table A4Estimated standard deviations of SVAR shocks

	Description		Prior		Post	erior	HPD interval		
		Distribu- tion	Mean	St. d.	Mean	St. d.	10%	90%	
$100\sigma_{\mu_z}$	Unit root technology	Inv-Γ	0.25	inf	0.328	0.052	0.248	0.406	
$100\sigma_{y^*}$	Foreign GDP	Inv-Γ	0.50	inf	0.317	0.055	0.219	0.415	
$1000\sigma_{\pi^{*}}$	Foreign inflation	Inv-Γ	0.50	inf	0.593	0.118	0.394	0.805	
$100\sigma_{R^*}$	Foreign interest rate	Inv-Γ	0.075	inf	0.067	0.008	0.054	0.079	

Note: Based on a single Metropolis-Hastings chain with 100 000 draws after a burn-in period of 900 000 draws.

Table A5Estimated parameters

	Parameter description	Prior			Posterior				HPD interval	
	···· r···	Distri-	Mean	St. d.	Mean St. d.				10% 90%	
		bution			finfric	full	finfric	full	fu	
ξ_d	Calvo, domestic	β	0.75	0.075	0.804	0.808	0.023	0.020	0.766	0.850
ξ_x	Calvo, exports	β	0.75	0.075	0.860	0.888	0.031	0.023	0.839	0.940
$\frac{\xi_x}{\xi_{mc}}$	Calvo, imports for	Ρ	0.75	0.075	0.000	0.000	0.051	0.025	0.057	0.910
Smc	consumption	β	0.75	0.075	0.779	0.796	0.049	0.053	0.712	0.879
ξ_{mi}	Calvo, imports for	P	0.70	0.070	0.775	0.770	0.017	0.000	0.712	0.077
Smi	investment	β	0.65	0.075	0.408	0.392	0.042	0.047	0.287	0.499
ξmx	Calvo, imports for exports	β	0.65	0.10	0.589	0.629	0.091	0.040	0.477	0.774
κ _d	Indexation, domestic	β	0.40	0.15	0.162	0.319	0.075	0.101	0.126	0.519
	Indexation, exports	β	0.40	0.15	0.301	0.367	0.107	0.098	0.120	0.573
κ_{χ}	Indexation, exports for	ρ	0.40	0.15	0.301	0.307	0.107	0.098	0.175	0.373
κ _{mc}	consumption	β	0.40	0.15	0.366	0.509	0.106	0.102	0.252	0.773
16	Indexation, imports for	μ	0.40	0.15	0.500	0.507	0.100	0.102	0.232	0.775
κ _{mi}	investment	β	0.40	0.15	0.249	0.304	0.100	0.102	0.102	0.528
K	Indexation, imports for	μ	0.40	0.15	0.247	0.504	0.100	0.102	0.102	0.520
κ_{mx}	exports	β	0.40	0.15	0.317	0.324	0.115	0.069	0.105	0.557
ĸ	Indexation, wages	β	0.40	0.15	0.241	0.233	0.079	0.083	0.081	0.398
$\frac{\kappa_w}{\nu^j}$	Working capital share	β	0.50	0.15	0.456	0.426	0.179	0.207	0.019	0.868
-	Inverse Frisch elasticity									
$0.1\sigma_L$		Г	0.30	0.15	0.287	0.965	0.106	0.113	0.610	1.317
b	Habit in consumption	β	0.65	0.15	0.898	0.878	0.030	0.030	0.801	0.946
0.1 <i>S''</i>	Investment adjustment costs	Г	0.50	0.15	0.168	0.179	0.030	0.037	0.089	0.260
σ_a	Variable capital utilisation	Г	0.20	0.075	0.567	0.365	0.093	0.058	0.181	0.547
η_x	Elasticity of substitution,									
	exports	Γ_{tr}	1.50	0.25	1.535	1.686	0.143	0.176	1.197	2.186
η_c	Elasticity of substitution,	-								
	consumption	Γ_{tr}	1.50	0.25	1.333	1.356	0.164	0.111	1.010	1.798
η_i	Elasticity of substitution,		1 50			1 0 (1		0.001	1 0 1 0	1 -00
	investment	Γ_{tr}	1.50	0.25	1.1*	1.261		0.091	1.010	1.598
η_f	Elasticity of substitution,		1 50	0.05	1 5 40	1 576	0.150	0.0.10	1 1 1 0	2 0 2 4
	foreign	Γ_{tr}	1.50	0.25	1.540	1.576	0.159	0.243	1.119	2.024
μ	Monitoring cost	β	0.30	0.075	0.273	0.256	0.040	0.033	0.184	0.333
hshare(%)	Hiring costs	Г	0.30	0.075		0.394		0.062	0.265	0.522
bshare	Utility flow, unemployed	β	0.75	0.075		0.799		0.038	0.708	0.883
F (%)	Endogenous separation rate	β	0.25	0.05		0.362		0.026	0.303	0.421
ρε	Persistence, stationary									
	technology	β	0.85	0.075	0.847	0.860	0.041	0.054	0.749	0.962
$ ho_{ m Y}$	Persistence, MEI	β	0.85	0.075	0.588	0.552	0.106	0.073	0.280	0.885
ρ_{ζ^c}	Persistence, consumption									
15	preferences	β	0.85	0.075	0.851	0.834	0.038	0.037	0.721	0.930
$ ho_{\zeta^h}$	Persistence, labour									
	preferences	β	0.85	0.075	0.817	0.958	0.048	0.019	0.922	0.991
$ ho_{\widetilde{\phi}}$	Persistence, country risk									
	premium	β	0.85	0.075	0.934	0.899	0.025	0.025	0.845	0.946
$ ho_g$	Persistence, government									
	spending	β	0.85	0.075	0.777	0.755	0.083	0.056	0.585	0.915
$ ho_{\gamma}$	Persistence, entrepreneurial	7								
	wealth	β	0.85	0.075	0.796	0.805	0.059	0.069	0.624	0.953

Notes: Based on two Metropolis–Hastings chains each with 100 000 draws after a burn-in period of 400 000 draws. * – calibrated in order to avoid numerical issues. Note that truncated priors, denoted by Γ_{tr} , with no mass below 1.01 have been used for the elasticity parameters η_j , $j = \{x, c, i, f\}$. "finfric" denotes benchmark financial frictions model, "full" stands for full model with unemployment.

	Description Prior				Poste	HPD interval				
		Distri-	Mean	St. d.	Me	an	St. d.		10%	90%
		bution			finfric full		finfric full		full	
$10\sigma_{\varepsilon}$	Stationary technology	Inv-Γ	0.15	inf	0.126	0.134	0.014	0.016	0.105	0.164
$\sigma_{ m Y}$	Marginal efficiency of investment	Inv-Γ	0.15	inf	0.157	0.170	0.027	0.036	0.065	0.262
σ_{ζ^c}	Consumption preferences	Inv-Γ	0.15	inf	0.236	0.207	0.056	0.046	0.116	0.320
σ_{ζ^h}	Labour preferences	Inv-Γ	0.50	inf	0.895	0.285	0.283	0.030	0.196	0.376
$100\sigma_{\widetilde{\phi}}$	Country risk premium	Inv-Γ	0.50	inf	0.552	0.551	0.045	0.048	0.461	0.651
$10\sigma_g$	Government spending	Inv-Γ	0.50	inf	0.471	0.469	0.041	0.043	0.382	0.564
$\sigma_{ au^d}$	Markup, domestic	Inv-Γ	0.50	inf	0.373	0.425	0.089	0.087	0.252	0.631
$\sigma_{\tau^{x}}$	Markup, exports	Inv-Γ	0.50	inf	0.992	1.635	0.391	0.605	0.581	3.111
$\sigma_{ au}^{m,c}$	Markup, imports for consumption	Inv-Γ	0.50	inf	0.863	0.775	0.329	0.271	0.260	1.433
$\sigma_{ au^{m,i}}$	Markup, imports for investment	Inv-Γ	0.50	inf	0.433	0.421	0.078	0.080	0.269	0.605
$\sigma_{\tau^{m,x}}$	Markup, imports for exports	Inv-Γ	0.50	inf	1.383	2.578	0.643	0.438	0.902	4.884
$10\sigma_{\gamma}$	Entrepreneurial wealth	Inv-Γ	0.50	inf	0.295	0.270	0.042	0.043	0.194	0.357

Table A6Estimated standard deviations of shocks

Notes: Based on two Metropolis–Hastings chains each with 100 000 draws after a burn-in period of 400 000 draws. "finfric" denotes benchmark financial frictions model, "full" stands for full model with unemployment.

A4. Data

The model is estimated using the data for Latvia (domestic part) and the euro area (foreign part). The sample period is Q1 1995–Q4 2012. 19 observable time series are used to estimate the model specification without vacancies. The same 19 variables plus first differenced vacancies are used for the second specification. The variables used in levels are nominal interest rate, GDP deflator inflation, CPI inflation, investment price index inflation, foreign CPI inflation, foreign nominal interest rate and interest rate spread. The rest of the variables are in terms of first differences of logs, and they are GDP, consumption, investment, exports, imports, government spending, real wage, real exchange rate, real stock prices, total hours worked, unemployment, and foreign GDP. The differenced variables are demeaned, except for total hours worked and unemployment. The domestic inflation rates and the real exchange rate are demeaned as well. All real quantities are in per capita terms.

A4.1 Posterior parameter values

The domestic and foreign blocks are estimated separately, since Latvia's economy has minuscule effect on the euro area economy⁷. The estimation results for the foreign SVAR model are obtained using a single Metropolis–Hastings chain with 100 000 draws after a burn-in of 900 000 draws. For the domestic block, the estimation results are obtained using two Metropolis–Hastings chains, each with

⁷ Latvia's share in the euro area is about 0.23% in terms of nominal GDP.

100 000 draws after a burn-in of 400 000 draws. Prior-posterior plots are relegated to Appendix B.

The posterior parameter estimates for the foreign block are reported in Tables A3 and A4, and those specific to the domestic block are given in Tables A5 and A6. For comparison, we also report the results for the domestic block of the benchmark financial frictions model.

The major differences in the estimated mean parameter values between the models are the following. First, the homogeneous domestic goods price indexation (to lagged inflation) parameter κ_d has moved closer to the prior mean, from 0.16 (benchmark model) to 0.32 (full model), resulting in a more rigid estimated homogeneous good price inflation.

The inverse Frisch elasticity parameter σ_L (which captures inverse elasticity of hours worked to the wage rate, given a constant marginal utility of wealth) has more than tripled, from 2.9 to 9.7 (above 7.7 reported by CTW for Sweden). This means that the estimated (non-inverted) Frisch elasticity has decreased from a rather standard level, from the US micro data perspective (Reichling and Whalen (2012)), of 0.34 to a rather low level of 0.1, indicating that employees vary their hours of work less in response to changes in their after-tax compensation.

The parameter governing the variable capital utilisation σ_a has decreased from 0.57 to 0.36, signaling more variation in capital utilisation. The persistence parameter governing labour preferences has increased from 0.82 to 0.96 and is the only persistence parameter whose posterior mean is above 0.9.

Also, and similar to CTW, the estimated standard deviation of the labour preference shock σ_{ζ^h} has decreased by a factor of three compared to the benchmark model. Thus, the model without the search and matching frictions relies on large amounts of high frequency variation of this shock to explain the data.⁸ See as well the graphical comparison of smoothed shocks in Appendix B.

Regarding the labour block, posterior mean of the utility flow parameter for the unemployed *bshare* is 0.80, above its prior mean (0.75), in line with the finding by Hagedorn and Manovskii (2008) that a high value of leisure helps fit the volatility of unemployment.

Hiring costs as a fraction of GDP are estimated to be 0.39%, which is higher than the prior mean (0.3) and about the same as reported by CTW for Sweden.

The endogenous separation rate is estimated to be 0.36%, up from its prior 0.25%, implying that about 10.7% of job separations are endogenous or cyclical, since the other part of separations, i.e. the exogenous separations, is fixed and, thus,

⁸ CTW interpretation of this difference is that the tight link between the desired real wage and total hours worked (through the marginal rate of substitution between leisure and consumption) implied by EHL labour market modelling does not hold in the data, even when this link is relaxed by assuming wage stickiness. This model instead implies efficient provision of labour on the intensive margin without any direct link to the (sticky, bargained) wage, and thereby allows for a high frequency disconnect between wages and hours worked. Fundamentally, as CTW note, this model reflects that labour is not supplied on a spot market but within long-term relationships.

acyclical.⁹ This endogenous separation rate is higher than approximately 6% reported in CTW for Sweden for the period Q1 1995–Q3 2010.

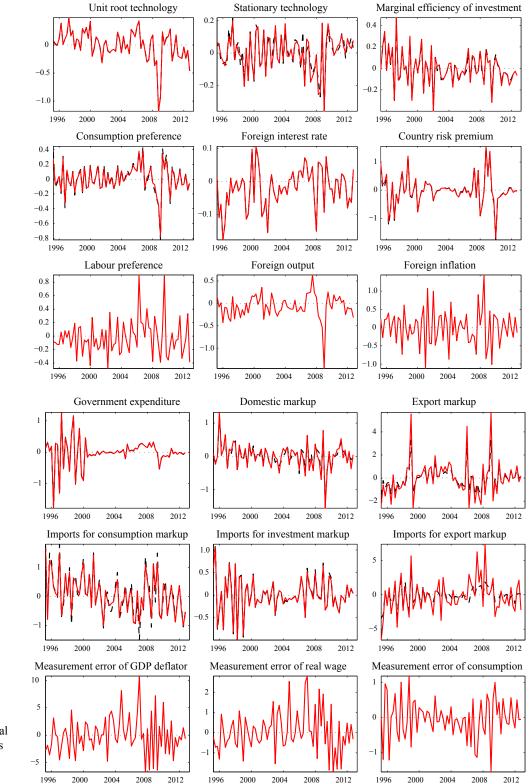
The bargaining power of workers η is solved to yield a steady state unemployment rate matching the sample average. The value of η at the posterior mean is 0.65, which is higher than 0.29 reported by CTW for Sweden and slightly higher than 0.5 suggested by conventional wisdom (Mortensen and Nagypál (2007)). This result may be due to the 2005-boom period in Latvia, during which several sectors of the economy experienced shortages of workers.

⁹ Author's unpublished results show that the share of the endogenous separation goes down to about 8%, if more generous data measurement errors are allowed than the current 10% of data variance.

Appendix B COMPUTATIONAL DETAILS

Figure B1

Smoothed shock processes and measurement errors



Financial frictions
 Full

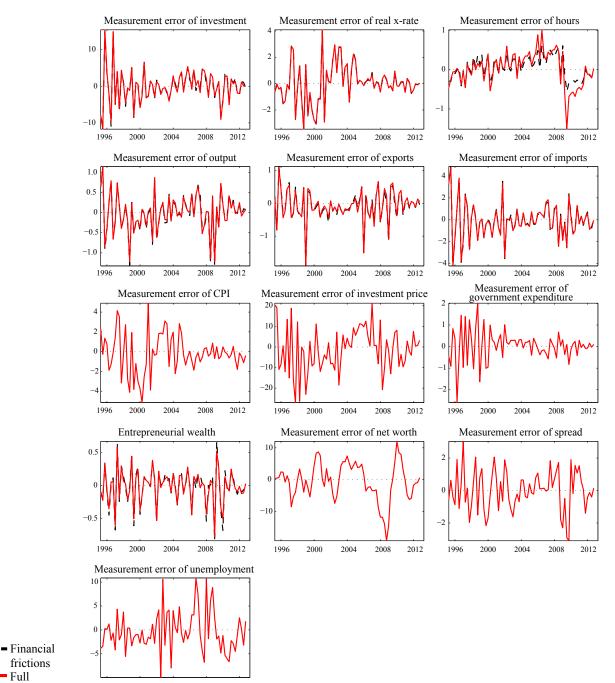


Figure B1 (cont.) **Smoothed shock processes and measurement errors**

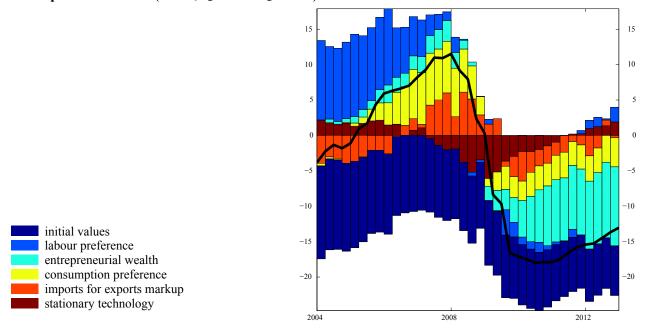


Figure B2 **Decomposition of GDP (levels; Q1 2004–Q4 2012)**

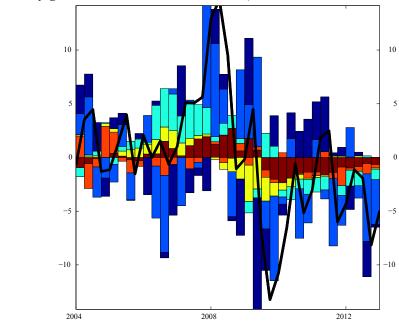
domestic markup

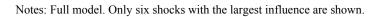
labour preference consumption preference measurement error of CPI stationary technology

imports for consumption markup

Notes: Full model. Only six shocks with the largest influence are shown.







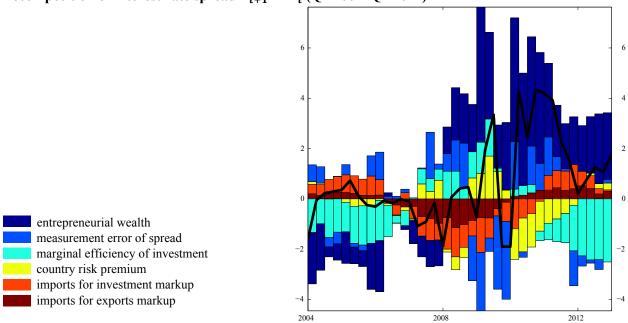
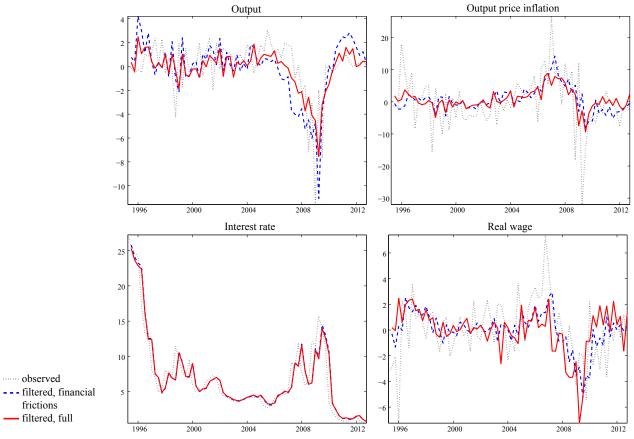


Figure B4 Decomposition of interest rate spread $Z_{t+1} - R_t$ (Q1 2004–Q4 2012)



Figure B5 One-step ahead forecasts



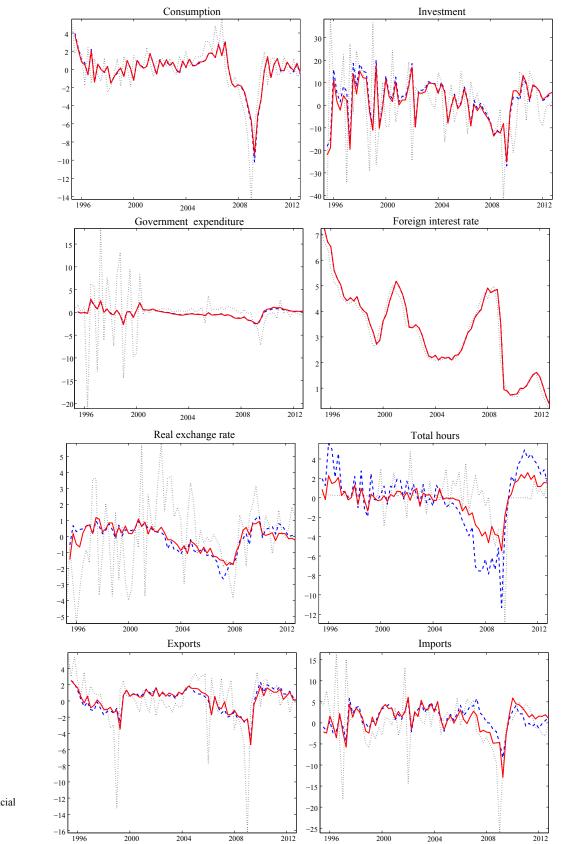


Figure B5 (cont.) **One-step ahead forecasts**

observed
 filtered, financial frictions
 filtered, full

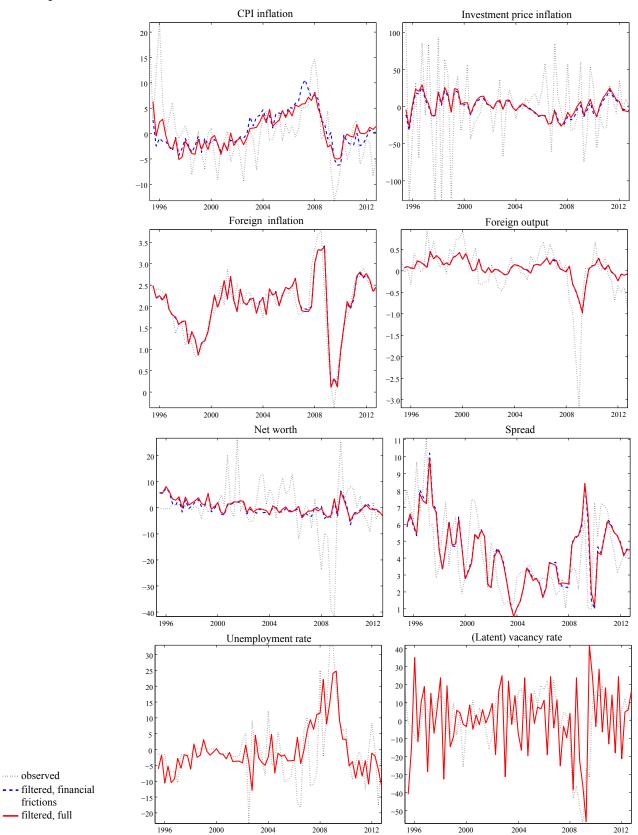
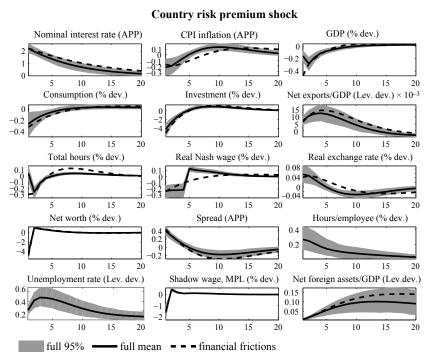


Figure B5 (cont.) **One-step ahead forecasts**

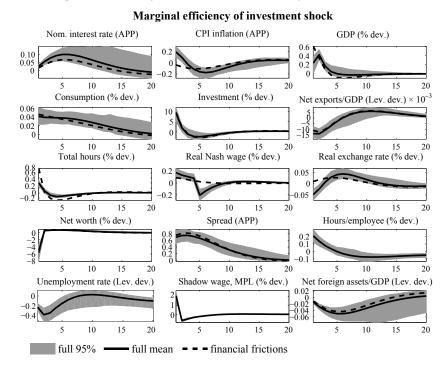


Figure B6 Impulse responses to country risk premium shock $\widetilde{\phi}_t$



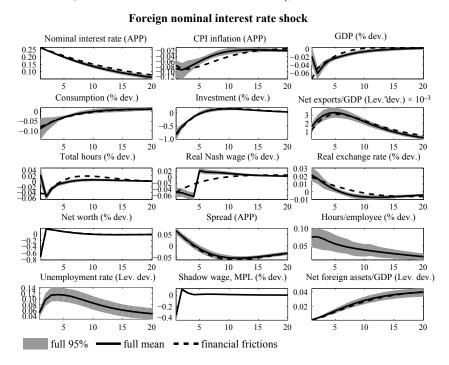
Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B7 Impulse responses to marginal efficiency of investment shock Υ_t



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

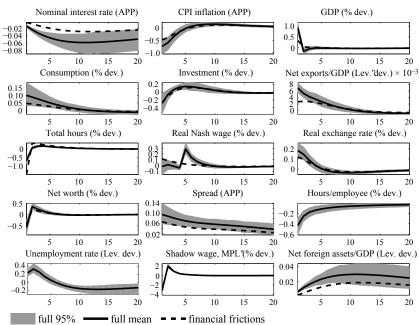
Figure B8 Impulse responses to foreign nominal interest rate shock $\varepsilon_{R^*,t}$



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

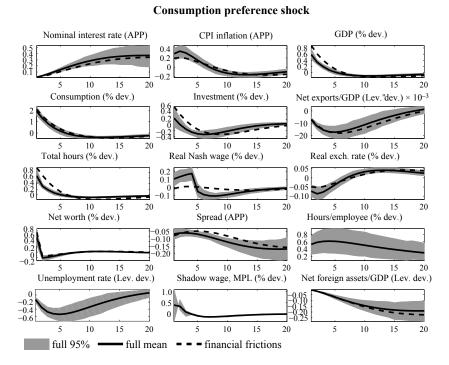
Figure B9 Impulse responses to stationary neutral technology shock ε_t

Stationary neutral technology shock



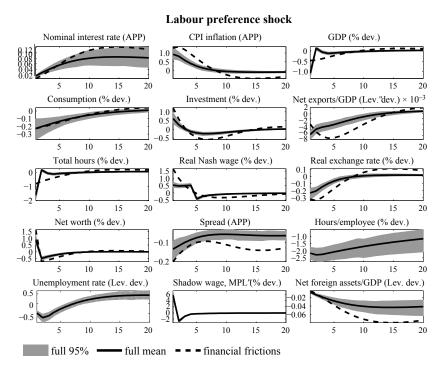
Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B10 Impulse responses to consumption preference shock ζ_t^c



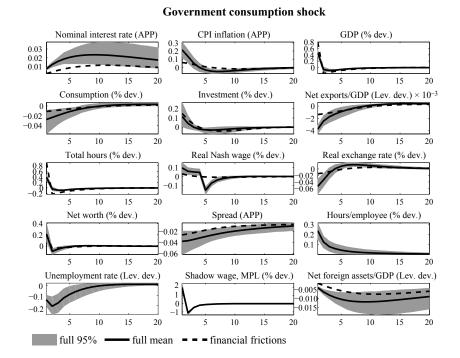
Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B11 Impulse responses to labour preference shock ζ_t^h



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B12 Impulse responses to government consumption shock g_t



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B13 Impulse responses to domestic markup shock τ_t^d

Domestic markup shock

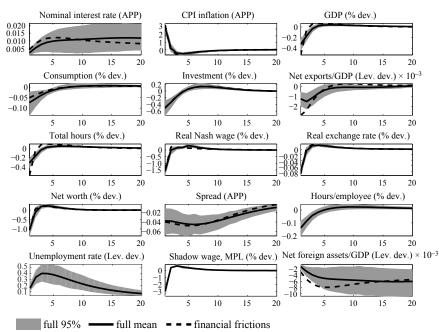
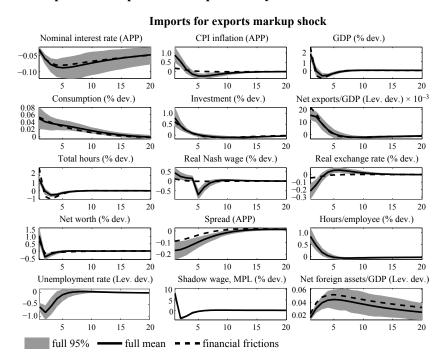


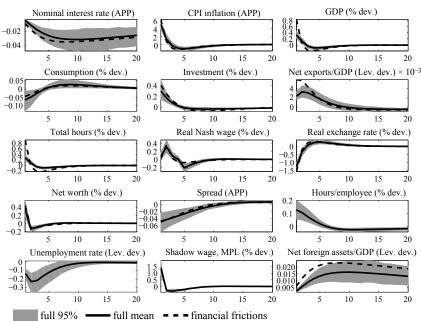
Figure B14 Impulse responses to imports for exports markup shock τ_t^{mx}



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B15 Impulse responses to imports for consumption markup shock τ_t^{mc}

Imports for consumption markup shock



Imports for investment markup shock

Figure B16 Impulse responses to imports for investment markup shock τ_t^{mi}

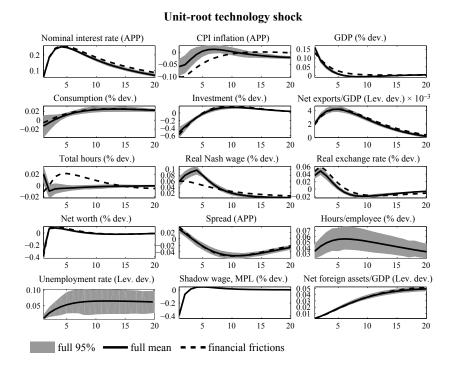
Nominal interest rate (APP) CPI inflation (APP) GDP (% dev.) 0.4 -0.02-0.04-0.060.5 Net exports/GDP (Lev. dev.) $\times 10^{-3}$ Investment (% dev.) Consumption (% dev.) 0.01 $\frac{15}{10}$ -0.01 Total hours (% dev.) Real Nash wage (% dev.) Real exchange rate (% dev.) -0.2-0.2 -0.3 -0.4Net worth (% dev.) Spread (APP) Hours/employee (% dev.) -0.2-0.4-0.6 -0.8 Unemployment rate (Lev. dev.) Shadow wage, MPL (% dev.) Net foreign assets/GDP (Lev. dev.) 0.02 -0.2 -0.4 full 95% full mean - - financial frictions

Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B17 Impulse responses to export markup shock τ_t^x

Export markup shock CPI inflation (APP) GDP (% dev.) Nominal interest rate (APP) 0.2 -0.02-0.1-0.2-0.3-04 -0.04Consumption (% dev.) Investment (% dev.) Net exports/GDP (Lev. dev.) $\times 10^{-3}$ 0.04 0.02 -0.2 -0.4 Total hours (% dev.) Real Nash wage (% dev.) Real exchange rate (% dev.) 0.08 0.06 0.02 -0.02 -0.2 -0.4 -0.6 0.1 Net worth (% dev.) Spread (APP) Hours/employee (% dev.) 0.2 0.06 0.04 0.02 -0.1 -0.2 -0.4 -0.2 Shadow wage, MPL (% dev.) Net foreign assets/GDP (Lev. dev.) $\times \, 10^{-3}$ Unemployment rate (Lev. dev.) 0.5 0.04 0.02 = full mean = = = financial frictions full 95% 🗕

Figure B18 Impulse responses to unit-root technology shock $\mu_{z,t}$



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B19 Impulse responses to foreign inflation shock $\varepsilon_{\pi^*,t}$

Foreign inflation shock

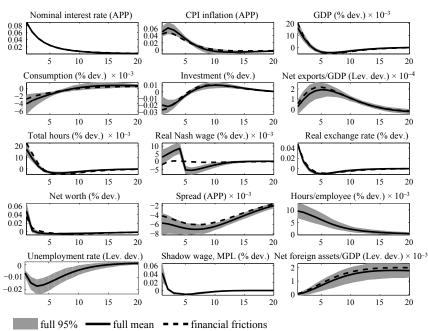
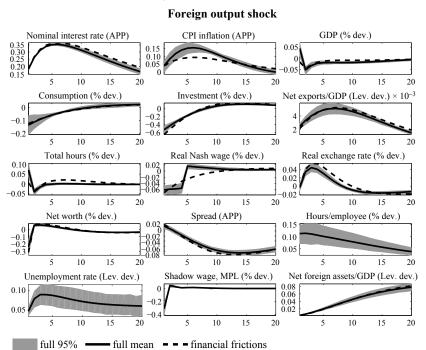
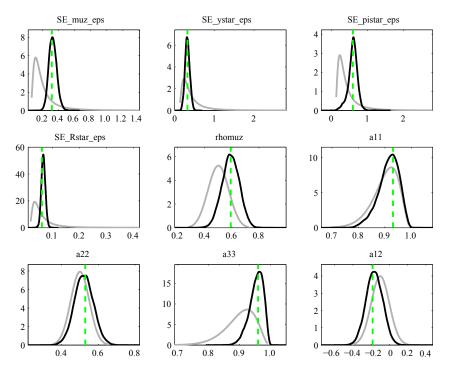


Figure B20 Impulse responses to foreign output shock $\varepsilon_{y^*,t}$



Note: The units on the y-axis are either in terms of percentage deviation (% dev.) from the steady state, annual percentage points (APP), or level deviation (Lev. dev.).

Figure B21 SVAR priors and posteriors



Note: Prior distribution is in gray, simulated distribution in black, and the computed posterior mode in dashed green.

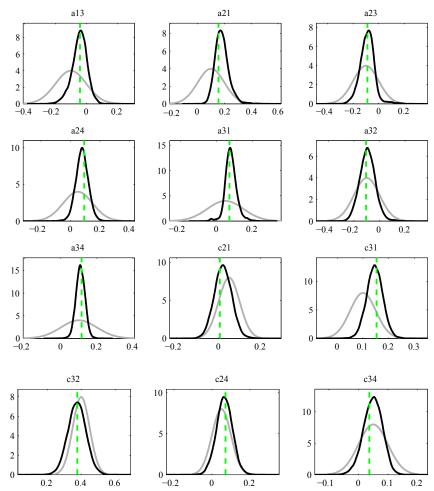
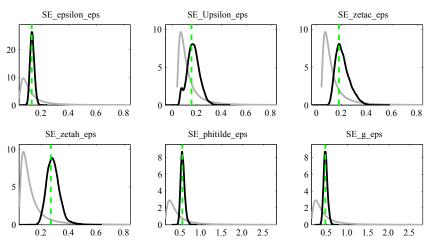


Figure B21 (cont.) **SVAR priors and posteriors**

Note: Prior distribution is in gray, simulated distribution in black, and the computed posterior mode in dashed green.

Figure B22 **Priors and posteriors**



Notes: Full model. Prior distribution is in gray, simulated distribution in black, and the computed posterior mode in dashed green.

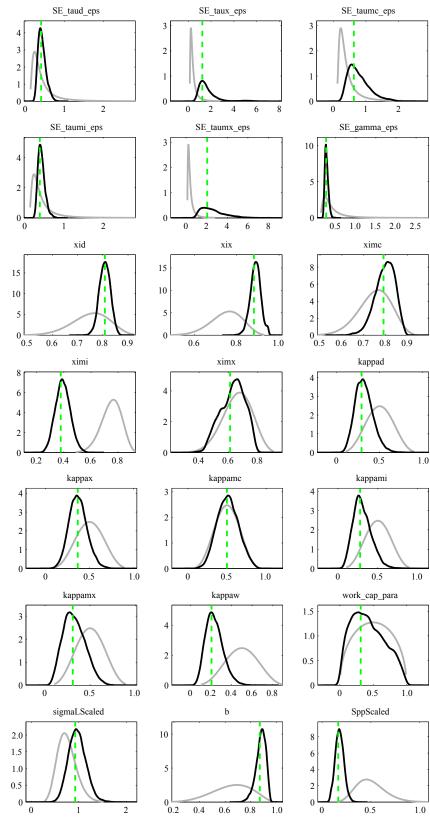


Figure B22 (cont.) **Priors and posteriors**

Notes: Full model. Prior distribution is in gray, simulated distribution in black, and the computed posterior mode in dashed green.

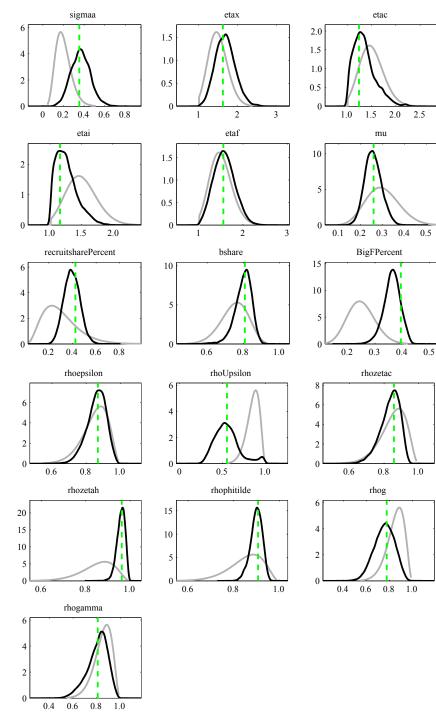


Figure B22 (cont.) **Priors and posteriors**

Notes: Full model. Prior distribution is in gray, simulated distribution in black, and the computed posterior mode in dashed green.

Appendix C MODEL DETAILS

To save space, this Section covers the details only about the labour block of the model. For the details on its core block and financial frictions block, see Appendices in Buss (2015) or CTW.

C1. Employment frictions block

This Section replaces the model of the labour market in the core block with the search and matching framework of Mortensen and Pissarides (1994), Hall (2005a, 2005b) and Shimer (2005, 2012) as implemented in CTW. Endogenous separation of employees from their jobs is allowed, as in, e.g. dHRW (2000). An implication of this modelling is the increased volatility in unemployment. Also, Taylor-type wage frictions are used instead of Calvo frictions due to the fact that empirically wage contracts normally have a fixed length and due to the ability to check that the wage always remains in the bargaining set in later periods of the wage contract.

C1.1 Sketch of the model

The model adopts the Dixit–Stiglitz specification of homogeneous goods production. A representative competitive retail firm aggregates differentiated intermediate goods into a homogeneous good. Intermediate goods are supplied by monopolists who hire labour and capital services in competitive factor markets. The intermediate goods firms are assumed to be subject to the same Calvo price setting friction as in the core block.

In the core block, the homogeneous labour services are supplied to the competitive labour market by labour contractors who combine labour services supplied to them by households that monopolistically supply specialised labour services. In this model, specialised labour services abstraction is not used. Instead, labour services are supplied by "employment agencies" to the homogeneous labour market where they are bought by intermediate goods producers. The change leaves the equilibrium conditions associated with the production of the homogeneous good unaffected. Key labour market activities, among them vacancy postings, layoffs, labour bargaining, setting the intensity of labour effort, are all carried out inside employment agencies.

Each household is composed of many workers, each of whom is in the labour force¹⁰. A worker begins the period either unemployed or employed with a particular employment agency. Unemployed workers do undirected search. They find a job with a particular agency with a probability that is proportional to the efforts made by the agency to attract workers. Workers are separated from employment agencies either exogenously or because they are actively cut. Workers pass back and forth between unemployment and employment, but there are no agency to agency transitions.

The events during the period in an employment agency take place in the following order. Each employment agency begins a period with a stock of workers. That stock

¹⁰ In reality, the participation rate is also changing. To take that into account, we have tried to adjust the data on unemployment rate by the participation rate before fitting the model. The results show that the difference between adjusted and unadjusted unemployment rates is rather small compared to the total variance of unemployment rate, so, for simplicity, we disregard the adjustment by the participation rate. CTW (2010) endogenise labour force participation.

is immediately reduced by exogenous separations and increased by new arrivals that reflect agency's recruiting efforts in the previous period. Then, the economy's aggregate shocks materialise.

At this point, each agency's wage is set. The bargaining arrangement is atomistic, so that each worker bargains separately with a representative of the employment agency. The agencies are allocated permanently into N equal-sized cohorts, and each period 1/N agencies establish a new wage by Nash bargaining. When a new wage is set, it evolves over the subsequent N - 1 periods according to (3) and (4):

$$W_{j,t+1} = \tilde{\pi}_{w,t+1} W_{j,t} \tag{3},$$

$$\tilde{\pi}_{w,t+1} = (\pi_t^c)^{\kappa_w} (\bar{\pi}_{t+1}^c)^{1-\kappa_w - \dot{\mathbf{u}}_w} (\breve{\pi})^{\dot{\mathbf{u}}_w} (\mu_z)^{\vartheta_w}$$
(4).

Wage negotiated in a given period covers all workers employed at an agency for each of the subsequent N - 1 periods, even those that will not arrive until later.

Next, each worker draws an idiosyncratic productivity shock. A cut-off level of productivity is determined, and workers with lower productivity are laid off. From a technical point of view this modelling is symmetric to the modelling of entrepreneurial idiosyncratic risk and bankruptcy. Two mechanisms by which the cut-off is determined are considered. One is based on the total surplus of a given worker and the other is based purely on the employment agency's interest.

After the endogenous layoff decision, the employment agency posts vacancies and the intensive margin of labour supply is chosen efficiently by equating the marginal value of labour services to the employment agency with the marginal cost of providing it by the household. At this point, the employment agency supplies labour to the labour market.

We now describe various labour market activities in greater detail. We begin with the decisions at the end of the period and work backwards to the bargaining problem because the bargaining problem internalises everything that comes after.

C1.2 Hours per worker

The intensive margin of labour supply is chosen to equate the value of labour services to the employment agency with the cost of providing it by the household. To explain it, consider the utility function of the household, which is a modified version of that of the benchmark model:

$$E_{t} \sum_{l=0}^{\infty} \beta^{l-t} \{ \zeta_{t+l}^{c} \log(C_{t+l} - bC_{t+l-1}) - \zeta_{t+l}^{h} A_{L} \Big[\sum_{i=0}^{N-1} \frac{(S_{i,t+l})^{1+\sigma_{L}}}{1+\sigma_{L}} \Big[1 - \mathcal{F} \big(\bar{a}_{t+l}^{i}; \sigma_{a,t+l} \big) \Big] l_{t+l}^{i} \Big] \}$$
(5)

where $i \in \{0, ..., N-1\}$ indexes the cohort to which the employment agency belongs. Index i = 0 corresponds to the cohort whose employment agency renegotiates the wage in the current period, i = 1 corresponds to the cohort that renegotiated in the previous period, and so on. Object l_t^i denotes the number of workers in cohort i after exogenous separations and new arrivals from unemployment have occurred. Let a_t^i denote the idiosyncratic productivity shock drawn by a worker in cohort i. Then, \bar{a}_t^i denotes the endogenously-determined cutoff such that all workers with $a_t^i < \bar{a}_t^i$ are laid off from the firm. Also, let

$$\mathcal{F}_t^i = \mathcal{F}(\bar{a}_t^i; \sigma_{a,t}) = \int_0^{\bar{a}_t^i} d\mathcal{F}(a; \sigma_{a,t})$$
(6)

denote the cumulative distribution function of idiosyncratic productivity. We assume that \mathcal{F} is lognormal with E(a) = 1 and $V(\log(a)) = \sigma_a^2$. Accordingly,

$$\left[1 - \mathcal{F}_t^i\right] l_t^i \tag{7}$$

denotes the number of workers with an employment agency in the *i*-th cohort who survive endogenous layoffs.

Let $\varsigma_{i,t}$ denote the number of hours supplied by a worker in the *i-th* cohort. The absence of the index *a* on $\varsigma_{i,t}$ reflects the assumption that each worker who survives endogenous layoffs in cohort *i* works the same number of hours, regardless of the realisation of their idiosyncratic level of productivity. One justification for this is that any connection between hours and idiosyncratic productivity might induce workers to manipulate real or perceived productivity downwards. The disutility experienced by a worker that works $\varsigma_{i,t}$ hours is

$$\zeta_t^h A_L \frac{(\varsigma_{i,t})^{1+\sigma_L}}{1+\sigma_L}$$

The household utility function (5) sums the disutility experienced by workers in each cohort.

Although the individual employed or unemployed worker's labour market experience is determined by idiosyncratic shocks, each household has sufficiently many workers so that the total fraction of workers employed

$$L_t = \sum_{i=0}^{N-1} \left[1 - \mathcal{F}_t^i\right] l_t^i$$

as well as the fractions allocated among different cohorts $[1 - \mathcal{F}_t^i]l_t^i$, i = 0, ..., N - 1 are the same for each household. It is assumed that all workers of a household are supplied inelastically to the labour market, i.e. labour force participation is constant.

Household's current receipts arising from the labour market are

$$(1 - \tau^{y})(1 - L_{t})P_{t}b^{u}z_{t}^{+} + \sum_{i=0}^{N-1} W_{t}^{i} [1 - \mathcal{F}_{t}^{i}]l_{t}^{i}\varsigma_{i,t} \frac{1 - \tau^{y}}{1 + \tau^{w}}$$

$$\tag{8}$$

where W_t^i is the rate of nominal wage earned by workers in cohort i = 0, ..., N - 1. The presence of the term involving b^u indicates the assumption that unemployed workers $1 - L_t$ receive a pre-tax payment of $b^u z_t^+$ final consumption goods. These unemployment benefits are financed by lump sum taxes. As in the core model, there is a labour income tax τ_v and a payroll tax τ^w that affect the after-tax wage.

Let W_t denote the price or "shadow wage" received by employment agencies for supplying one unit of (effective) labour service to intermediate goods producers. It represents the marginal gain of the employment agency that occurs when an individual worker increases the time spent working by one (effective) unit. As the employment agency is competitive in the supply of labour services, it takes W_t as given, and in equilibrium it coincides with the marginal product of labor and is connected to the marginal cost of intermediate goods producers through (9) and (10):

$$mc_t = \tau_t^d \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \left(\frac{1}{\alpha}\right)^{\alpha} (r_t^k)^{\alpha} \left(\overline{w}_t R_t^f\right)^{1-\alpha} \frac{1}{\varepsilon_t},\tag{9},$$

$$mc_t = \tau_t^d \frac{(\mu_{\Psi,t})^{\alpha} \overline{w}_t R'_t}{\varepsilon_t (1-\alpha) \left(\frac{k_{i,t}}{\mu_z + t^{H_{i,t}}}\right)^{\alpha}}$$
(10).

A real world interpretation is that it is the shadow value of an extra hour of work supplied by the human resources department to a firm.

It is assumed that hours per worker are chosen to equate the worker's marginal cost of working with the agency's marginal benefit:

$$W_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{v_t \frac{1-\tau^y}{1+\tau^w}}$$
(11)

for i = 0, ..., N - 1, where \mathcal{G}_t^i de s expected productivity of workers who survive endogenous separation:

$$\mathcal{G}_t^i = \frac{\mathcal{E}_t^i}{1 - \mathcal{F}_t^i} \tag{12}$$

where

$$\mathcal{E}_t^i := \mathcal{E}(\bar{a}_t^i; \sigma_{a,t}) := \int_{\bar{a}_t^i}^{\infty} a d\mathcal{F}(a; \sigma_{a,t})$$
(13).

To understand the expression on the right-side of (11), note that the marginal cost, in utility terms, of an individual worker who increases hours worked by one unit is $\zeta_t^h A_L \zeta_{i,t}^{\sigma_L}$. This is converted to currency units by dividing by the multiplier v_t (on the household's nominal budget constraint) and the tax wedge $(1 - \tau^{\gamma})/(1 + \tau^{w})$. The left side of (11) represents the increase in revenues of the employment agency from increasing hours worked by one unit. Division by $1 - \mathcal{F}_t^i$ is required in (12) so that the expectation is relative to the distribution of *a* conditional on $a > \overline{a}_t^j$.

Labour intensity is potentially different across cohorts because G_t^i is indexed by cohort. When the wage rate is determined by Nash bargaining, it is taken into account that labour intensity is calculated according to (11) and that workers will endogenously separate. Note that labour intensity as determined by (11) is efficient and unaffected by the negotiated wage and its rigidity.

C1.3 Vacancies and employment agency problem

The employment agency in the *i*-th cohort determines how many employees it will have in period t + 1 by choosing vacancies v_t^i . The costs associated with v_t^i are

$$\frac{\kappa z_t^+}{\varphi} \Big(\frac{Q_t^i v_t^i}{[1 - \mathcal{F}_t^i] l_t^i} \Big)^{\varphi} \left[1 - \mathcal{F}_t^i \right] l_t^i$$

units of the domestic homogeneous good. Parameter $\varphi > 1$ determines the curvature of the cost function. Convex costs of adjusting the work force are assumed because linear costs would imply indeterminacy, as dynamic wage dispersion implies that the costs of employees are heterogeneous across agencies, while the benefit of an additional employee is the same across agencies. $\kappa z_t^+/\varphi$ is a cost parameter, which is assumed to grow at the same rate as the overall economy, and, as noted above, $[1 - \mathcal{F}_t^i]l_t^i$ denotes the number of employees in the *i*-th cohort after endogenous separations have occurred. Also, Q_t is the probability that a posted vacancy is filled, a quantity that is exogenous to an individual employment agency. If $\iota = 1$, costs are a function of the number of people hired, not the number of vacancy postings. Thus, $\iota = 1$ emphasises the internal costs (e.g. for training) of adjusting the work force but not the search costs. [Consider a shock that triggers an economic expansion and also produces a fall in the probability of filling a vacancy Q_t . Then the expansion will be smaller in the version of the model that emphasises search costs ($\iota = 0$) than in the version that emphasises internal costs ($\iota = 1$).]

To further describe the vacancy decisions of employment agencies, their objective function is required. We begin by considering $F(l_t^0, \omega_t)$, the value function of a representative employment agency in cohort i = 0 that negotiates its wage in the current period. The arguments of F are agency's workforce after beginning-of-period exogenous separations and new arrivals l_t^0 , and an arbitrary value of the nominal wage rate ω_t . In other words, we consider the value of the firm's problem after the wage rate has been set.

We assume that the firm chooses a particular monotone transform of vacancy postings denoted by \tilde{v}_t^i :

$$\tilde{v}_t^i := \frac{Q_t^i v_t^i}{(1 - \mathcal{F}_t^j) l_t^i}.$$

The agency's hiring rate χ_t^i is related to \tilde{v}_t^i by

$$\chi_t^i = Q_t^{1-\iota} \tilde{v}_t^i \tag{14}.$$

To construct $F(l_t^0, \omega_t)$, one needs to derive the law of motion of the firm's workforce during the period of wage contract. Time t + 1 workforce of the representative agency in the *i*-th cohort at time t is denoted by l_{t+1}^{i+1} . That workforce reflects endogenous separations in period t as well as exogenous separations and new arrivals at the beginning of period t + 1. Let ρ denote the probability that an individual worker attached to an employment agency at the beginning of a period survives the exogenous separation. Then, given the hiring rate χ_t^i , we derive that

$$l_{t+1}^{j+1} = (\chi_t^j + \rho)(1 - \mathcal{F}_t^j)l_t^j$$
(15)

for j = 0, 1, ..., N - 1, with the understanding that j = N is to be interpreted as j = 0.

The value function of the firm is

$$\begin{split} F(l_{t}^{0},\omega_{t}) &= \sum_{j=0}^{N-1} \beta^{j} E_{t} \frac{v_{t+j}}{v_{t}} \max_{(\tilde{v}_{t+j}^{i}, \bar{a}_{t+j}^{j})} [\int_{\bar{a}_{t+j}^{j}}^{\infty} (W_{t+j}a - \Gamma_{t,j}\omega_{t})\varsigma_{j,t+j}d\mathcal{F}(a) \\ &- P_{t+j} \frac{\kappa z_{t+j}^{+}}{\varphi} (\tilde{v}_{t+j}^{j})^{\varphi} (1 - \mathcal{F}_{t+j}^{j})] l_{t+j}^{j} \\ &+ \beta^{N} E_{t} \frac{v_{t+N}}{v_{t}} F(l_{t+N}^{0}, \widetilde{W}_{t+N}), \end{split}$$
(16)

where l_t^j evolves according to (15), $\varsigma_{j,t}$ satisfies (11) and

$$\Gamma_{t,j} = \begin{cases} \tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}, & j > 0\\ 1 & j = 0 \end{cases}$$
(17)

where $\tilde{\pi}_{w,t}$ has been defined in (4). Note that W_{t+j} denotes the price paid to the employment agency for supplying one unit of labour to intermediate goods producers in period t + j. Term $\Gamma_{t,j}\omega_t$ represents the wage rate in period t + j, given that the wage rate was ω_t at time t and there have been no wage negotiations in periods t + 1, t + 2, \cdots , t + j. In (16), \widetilde{W}_{t+N} denotes the Nash bargaining wage that is negotiated in period t + N, which is when the next round of bargaining occurs. At time t, the agency takes the state t + N-contingent function \widetilde{W}_{t+N} as given. Vacancy decisions of employment agencies solve the maximisation problem in (16).

From (16), $F(l_t^0, \omega_t)$ is linear in l_t^0 :

$$F(l_t^0, \omega_t) = J(\omega_t) l_t^0 \tag{18}$$

where $J(\omega_t)$ is not a function of l_t^0 and is the surplus that a firm bargaining in the current period enjoys from a match with an individual worker when the current wage is ω_t . Although later in the period workers become heterogeneous when they draw idiosyncratic shocks to productivity, the fact that the draw is i.i.d. over time means that workers are all identical at the time when (18) is evaluated.

C1.4 Worker value functions

In order to discuss the endogenous separation decision as well as the bargaining problem, we must have the value function of the individual worker. For the bargaining problem, we require the worker's value function before he knows what his idiosyncratic productivity draw is. For the endogenous separation problem, we need to know the worker's value function after he knows that he has survived the endogenous separation. For both bargaining and separation problems, we need to know the value of unemployment to the worker.

Let V_t^i denote the period t value of being a worker in an agency in cohort i after this worker survives that period's endogenous separation:

$$V_{t}^{i} = \Gamma_{t-i,i} \widetilde{W}_{t-i} \varsigma_{i,t} \frac{1-\tau^{y}}{1+\tau^{w}} - A_{L} \frac{\zeta_{t}^{h} \varsigma_{i,t}^{1+\sigma_{l}}}{(1+\sigma_{l})v_{t}} + \beta E_{t} \frac{v_{t+1}}{v_{t}} \Big[\rho (1 - \mathcal{F}_{t+1}^{i+1}) V_{t+1}^{i+1} + (1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}) U_{t+1} \Big]$$
(19)

for i = 0, 1, ..., N - 1, where \widetilde{W}_{t-i} denotes the wage negotiated *i* periods in the past, and $\Gamma_{t-i,i}\widetilde{W}_{t-i}$ represents the wage received in period *t* by workers in cohort *i*. The two terms after the equality in (19) represent a worker's period *t* flow utility, converted to units of currency. The term in square brackets in (19) corresponds to utility in the possible period t + 1 states in the world. With probability $\rho(1 - \mathcal{F}_{t+1}^{i+1})$, the worker survives the exogenous and endogenous separations in period t + 1, in which case its value function in t + 1 is V_{t+1}^{i+1} . With complementary probability, $1 - \rho(1 - \mathcal{F}_{t+1}^{i+1})$, the worker separates into unemployment in period t + 1 and enjoys utility U_{t+1} .

The currency value of being unemployed in period t is

$$U_t = P_t z_t^+ b^u (1 - \tau^y) + \beta E_t \frac{v_{t+1}}{v_t} [f_t V_{t+1}^x + (1 - f_t) U_{t+1}]$$
(20)

where f_t is the probability that an unemployed worker will land a job in period t + 1, V_{t+1}^x is the period t + 1 value function of a worker who knows that he has matched with an employment agency at the beginning of t + 1 but does not know which one. In particular,

$$V_{t+1}^{x} = \sum_{i=0}^{N-1} \frac{\chi_{t}^{i} (1-\mathcal{F}_{t}^{i}) l_{t}^{i}}{m_{t}} \tilde{V}_{t+1}^{i+1}$$
(21)

where total new matches m_t at the beginning of period t + 1, is given by

$$m_t = \sum_{j=0}^{N-1} \chi_t^j (1 - \mathcal{F}_t^j) l_t^j$$
(22).

In (21), $\chi_t^i (1 - \mathcal{F}_t^i) l_t^i / m_t$ is the probability of finding a job in period t + 1 in an agency belonging to cohort *i* in period *t*. This is a proper probability distribution, because it is positive for each *i*, and it sums to unity by (22).

In (21), \tilde{V}_{t+1}^{i+1} is the analogue of V_{t+1}^{i+1} , except that the former is defined before the worker knows if he has survived the endogenous productivity cut, while the latter is defined after survival. The superscript i + 1 appears on \tilde{V}_{t+1}^{i+1} , because probabilities in (21) refer to activities in a particular agency cohort in period t, while in period t + 1, the index of that cohort is incremented by unity.

The definition of U_t in (20) is completed by giving the formal definition of \tilde{V}_t^{j} :

$$\tilde{V}_t^j = \mathcal{F}_t^j U_t + (1 - \mathcal{F}_t^j) V_t^j$$
(23),

that is, at the beginning of the period, the worker has probability \mathcal{F}_t^j of returning to unemployment and complementary probability of surviving in the firm, i.e. to work and receive a wage in period t.

C1.5 Separation decision

Here we discuss the separation decision of a representative agency in j = 0 cohort which renegotiates the wage in the current period. Decisions of the other cohorts are made in a similar way. Just prior to the realisation of idiosyncratic worker uncertainty, the number of workers attached to the representative agency in j = 0cohort is l_t^0 . Each worker in l_t^0 independently draws productivity a from the cumulative distribution function \mathcal{F} . Workers who draw a value of a below the productivity cut-off \bar{a}_t^0 are separated from the agency, while the rest remain. The productivity cut-off is selected by the representative agency taking as given all variables determined outside the agency. Alternative criteria for selecting \bar{a}_t^0 are considered. The different criteria correspond to different ways of weighting the surplus enjoyed by the agency and the surplus enjoyed by workers l_t^0 attached to the agency.

The aggregate surplus across all the l_t^0 workers in the representative agency is given by

$$(V_t^0 - U_t)(1 - \mathcal{F}t^0)l_t^0 \tag{24}.$$

Note that each worker in fraction $1 - \mathcal{F}_t^0$ of workers with $a \ge \bar{a}_t^0$, who stay with the agency, experiences the same surplus $V_t^0 - U_t$. Fraction \mathcal{F}_t^0 of workers in l_t^0 , who leave, enjoys zero surplus. Object \mathcal{F}_t^0 is a function of \bar{a}_t^0 as indicated in (6).

The surplus enjoyed by the representative employment agency before idiosyncratic worker uncertainty is realised and when the workforce is l_t^0 , is given by (16). According to (18), agency surplus per worker in l_t^0 is given by $J(\omega_t)$ having the following structure:

$$J(\omega_t) = \max_{\bar{a}_t^0} \tilde{J}(\omega_t; \bar{a}_t^0) (1 - \mathcal{F}_t^0)$$

where

$$\tilde{J}(\omega_t; \bar{a}_t^0) = \max_{\tilde{v}_t^0} \left\{ (W_t \mathcal{G}_t^0 - \omega_t) \varsigma_{0,t} - P_t z_t^+ \frac{\kappa}{\varphi} (\tilde{v}_t^0)^\varphi + \beta \frac{v_{t+1}}{v_t} (\chi_t^0 + \rho) J_{t+1}^1(\omega_t) \right\} (25)$$

denotes the value to an agency in cohort 0 of an employee after endogenous separation has taken place. Terms χ_t^0 and \tilde{v}_t^0 are connected by (14). Thus, the surplus of the representative agency with workforce l_t^0 expressed as a function of an arbitrary value of \bar{a}_t^0 is

$$\tilde{J}(\omega_t; \bar{a}_t^0)(1 - \mathcal{F}_t^0)l_t^0.$$
⁽²⁶⁾

This expression displays the two ways in which \bar{a}_t^0 impacts on firm profits: \bar{a}_t^0 affects the number of workers $1 - \mathcal{F}_t^0$ employed in period t and their average productivity, thereby affecting the value to the employer of an employee \tilde{J} . The impact of \bar{a}_t^0 on the number of workers can be deduced from (6). Although at first glance it may appear that the cut-off affects \tilde{J} in several ways, in fact it affects \tilde{J} only through the above two channels.

The surplus criterion governing the choice of \bar{a}_t^0 is specified to be a weighted sum of the worker surplus and the employer surplus described above:

$$[s_w(V_t^0 - U_t) + s_e \tilde{J}(\omega_t; \bar{a}_t^0)](1 - \mathcal{F}_t^0)l_t^0$$
(27)

where parameters $s_w, s_e \in \{0,1\}$ allow for a variety of surplus measures. If $s_w = 0$ and $s_e = 1$, we have the employer surplus. If $s_w = s_e = 1$, we have the total surplus. Accordingly, the employer surplus model is the one in which \bar{a}_t^0 is chosen to optimise (27) with $s_w = 0$, $s_e = 1$, and the total surplus model is the one that optimises (27) with $s_w = s_e = 1$. The first order condition for an interior optimum is

$$s_{w}V_{t}^{0\prime} + s_{e}\tilde{J}_{\bar{a}^{0}}(\omega_{t};\bar{a}_{t}^{0}) = [s_{w}(V_{t}^{0} - U_{t}) + s_{e}\tilde{J}(\omega_{t};\bar{a}_{t}^{0})]\frac{\mathcal{F}_{t}^{0\prime}}{1 - \mathcal{F}_{t}^{0}}$$
(28),

according to which \bar{a}_t^0 is selected to balance the impact on surplus along intensive and extensive margins. The expression on the left side of the equality characterises the impact on the intensive margin: the surplus per worker who survives the cut increases with \bar{a}_t^0 . The expression on the right side of (28) captures the extensive margin, the loss of surplus associated with $\mathcal{F}_t^{0'}/(1-\mathcal{F}_t^0)$ workers who do not survive the cut. The equations that characterise the choice of \bar{a}_t^j , j = 1, ..., N - 1 are essentially the same as (28).

Expression (28) assumes an arbitrary wage outcome ω_t . Next, we discuss the bargaining problem that determines the value for ω_t .

C1.6 Bargaining problem

Bargaining occurs among a continuum of worker–agency representative pairs. Each bargaining session takes the outcomes of all other bargaining session as given. As bargaining sessions are atomistic, each of them ignores its impact on the wage earned by workers arriving in the future during the contract. It is assumed that those future workers are simply paid the average of the outcome of all bargaining sessions. Since all bargaining problems are identical, the wage that solves each problem is the same, and so the average wage coincides with the wage that solves the individual bargaining problem. Since every bargaining session is atomistic, it also ignores the impact of wage bargaining on decisions – like vacancies and separations – taken by the firm.

The Nash bargaining problem that determines the wage rate is a combination of the worker surplus and the firm surplus:

$$\max_{\omega_t} \left(\tilde{V}_t^0 - U_t \right)^{\eta} J(\omega_t)^{1-\eta}$$

where the firm surplus $J(\omega_t)$ reflects that the outside option of the firm in the bargaining problem is zero. The wage that solves this problem is denoted by \widetilde{W}_t .

Until now, it was explicitly assumed that the negotiated wage paid by an employment agency, which has renegotiated most recently in *i* periods in the past, is always inside the bargaining set $[\underline{w}_t^i, \overline{w}_t^i]$, i = 0, 1, ..., N - 1. In other words, the wage paid is not lower than the workers reservation wage and not higher than the wage an employment agency is willing to pay. The fact that we allow for endogenous separation, when either total surplus or employer surplus of a match is negative, does not strictly guarantee that wages are in the bargaining set, i.e. that both the employer and the worker have a non-negative surplus of the match. This completes the description of the employment friction representation of the labour market. This block also brings three new shocks η_t , $\sigma_{m,t}$ and $\sigma_{a,t}$ into the model.

C2. Scaling of variables and functional forms

We adopt the following scaling of variables. The neutral shock to technology is z_t , and its growth rate is $\mu_{z,t}$:

$$\frac{z_t}{z_{t-1}} = \mu_{z,t}.$$

Variable Ψ_t is an investment-specific shock to technology, and it is convenient to define the following combination of investment-specific and neutral technology:

$$z_t^+ = \Psi_t^{\frac{\alpha}{1-\alpha}} z_t,$$

$$\mu_{z^+,t} = \mu_{\Psi,t}^{\frac{\alpha}{1-\alpha}} \mu_{z,t}$$
(29).

Capital \overline{K}_t and investment I_t are scaled by $z_t^+ \Psi_t$. Foreign and domestic inputs in production of I_t (we denote these by I_t^d and I_t^d respectively) are scaled by z_t^+ . Consumption goods (C_t^m are imported intermediate consumption goods, C_t^d are domestically produced intermediate consumption goods, and C_t are final consumption goods) are scaled by z_t^+ . Government spending, real wage and real foreign assets are scaled by z_t^+ . Exports (X_t^m are imported intermediate goods for

use in producing exports and X_t are final export goods) are scaled by z_t^+ . Also, v_t is the shadow value in utility terms to the household of domestic currency and $v_t P_t$ is the shadow value of one unit of the homogeneous domestic good. The latter must be multiplied by z_t^+ to induce stationarity. \tilde{P}_t is the within-sector relative price of a good. w_t denotes the ratio between the (Nash) wage paid to workers \tilde{W}_t and the "shadow wage" W_t paid by intermediate goods producers to employment agencies in the employment friction block. Thus:

$$\begin{aligned} k_{t+1} &= \frac{K_{t+1}}{z_t^+ \Psi_t}, \bar{k}_{t+1} = \frac{\bar{k}_{t+1}}{z_t^+ \Psi_t}, i_t^d = \frac{l_t^a}{z_t^+}, i_t = \frac{l_t}{z_t^+ \Psi_t}, i_m^t = \frac{l_t^m}{z_t^+}, \\ c_t^m &= \frac{c_t^m}{z_t^+}, c_t^d = \frac{c_t^d}{z_t^+}, c_t = \frac{c_t}{z_t^+}, g_t = \frac{g_t}{z_t^+}, \bar{w}_t = \frac{W_t}{z_t^+ P_t}, a_t := \frac{S_t A_{t+1}^*}{z_t^+ P_t}, \\ x_t^m &= \frac{X_t^m}{z_t^+}, x_t = \frac{X_t}{z_t^+}, \psi_{z^+,t} = v_t P_t z_t^+, (y_t =) \tilde{y}_t = \frac{Y_t}{z_t^+}, \tilde{p}_t = \frac{\tilde{P}_t}{P_t}, w_t = \frac{\tilde{w}_t}{W_t}, \\ n_{t+1} &= \frac{\bar{N}_{t+1}}{z_t^+ P_t}, w^e = \frac{W_t^e}{z_t^+ P_t}. \end{aligned}$$

We define the scaled date t price of new installed physical capital for the start of period t + 1 as $p_{k',t}$ and the scaled real rental rate of capital as \bar{r}_t^k :

$$p_{k',t} = \Psi_t P_{k',t}, \bar{r}_t^k = \Psi_t r_t^k$$

where $P_{k',t}$ is in units of the domestic homogeneous good.

The nominal exchange rate is denoted by S_t and its growth rate is s_t :

$$s_t = \frac{s_t}{s_{t-1}}.$$

We define the following inflation rates:

$$\pi_{t} = \frac{P_{t}}{P_{t-1}}, \pi_{t}^{c} = \frac{P_{t}^{c}}{P_{t-1}^{c}}, \pi_{t}^{*} = \frac{P_{t}^{*}}{P_{t-1}^{*}},$$
$$\pi_{t}^{i} = \frac{P_{t}^{i}}{P_{t-1}^{i}}, \pi_{t}^{\chi} = \frac{P_{t}^{\chi}}{P_{t-1}^{\chi}}, \pi_{t}^{m,j} = \frac{P_{t}^{m,j}}{P_{t-1}^{m,j}}$$

for j = c, x, i. Here, P_t is the price of a domestic homogeneous output good, P_t^c is the price of domestic final consumption goods (i.e. the CPI), P_t^* is the price of a foreign homogeneous good, P_t^i is the price of the domestic final investment good, and P_t^x is the price (in foreign currency units) of a final export good.

With one exception, we define a lower case price as the corresponding uppercase price divided by the price of the homogeneous good. When the price is denominated in domestic currency units, we divide by the price of the domestic homogeneous good, P_t . When the price is denominated in foreign currency units, we divide by P_t^* , the price of the foreign homogeneous good. The exceptional case has to do with handling of the price of investment goods P_t^i . It grows at a rate slower than P_t , and we therefore scale it by P_t/Ψ_t . Thus:

$$p_t^{m,x} = \frac{P_t^{m,x}}{P_t}, p_t^{m,c} = \frac{P_t^{m,c}}{P_t}, p_t^{m,i} = \frac{P_t^{m,i}}{P_t},$$

$$p_t^{\chi} = \frac{P_t^{\chi}}{P_t^{\chi}}, p_t^{c} = \frac{P_t^{c}}{P_t}, p_t^{i} = \frac{\Psi_t P_t^{i}}{P_t}$$
(30).

Here, m, j means the price of an imported good which is subsequently used in the production of exports in the case of j = x, in the production of the final consumption good in the case of j = c, and in the production of final investment good in the case of j = i. When there is just a single superscript, the underlying good is a final good, with j = x, c, i corresponding to exports, consumption and investment respectively.

Functional forms

In the employment friction block we assume a log-normal distribution for idiosyncratic productivities of workers. This implies the following:

$$\mathcal{E}(\bar{a}_t^j;\sigma_{a,t}) = \int_{\bar{a}_t^j}^{\infty} ad\mathcal{F}(a;\sigma_{a,t}) = 1 - prob\left[v < \frac{\log(\bar{a}_t^j) + \frac{1}{2}\sigma_{a,t}^2}{\sigma_{a,t}} - \sigma_{a,t}\right]$$
(31)

where *prob* refers to standard normal distribution, and (31) simply is (13) spelled out under this distributional assumption. Similarly, equation (6) becomes

$$\mathcal{F}(\bar{a}^{j};\sigma_{a}) = \int_{0}^{\bar{a}^{j}} d\mathcal{F}(a;\sigma_{a}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log(\bar{a}^{j}) + \frac{1}{2}\sigma_{a}^{2}} \exp\left(\frac{-v^{2}}{2}\right) dv$$
$$= prob\left[v < \frac{\log(\bar{a}^{j}) + \frac{1}{2}\sigma_{a}^{2}}{\sigma_{a}}\right]$$
(32).

C3. Equilibrium conditions for employment frictions block

C3.1 Labour hours

Scaling (11) by $P_t z_t^+$ yields

$$\overline{w}_t \mathcal{G}_t^i = \zeta_t^h A_L \varsigma_{i,t}^{\sigma_L} \frac{1}{\psi_{z^+, t^{\frac{1-\tau^y}{1+\tau^w}}}}$$
(33).

Note that the ratio

$$\frac{\mathcal{G}_t^\iota}{\varsigma_{i,t}^{\sigma_L}}$$

will be the same for all cohorts since no other variables in (33) are indexed by cohort.

C3.2 Vacancies and employment agency problem

An employment agency in the *i-th* cohort which does not renegotiate its wage in period t, sets the period t wage $W_{i,t}$ as in (3):

$$W_{i,t} = \tilde{\pi}_{w,t} W_{i-1,t-1}, \tilde{\pi}_{w,t} := (\pi_{t-1})^{\kappa_w} (\bar{\pi}_t)^{1-\kappa_w - \dot{u}_w} (\breve{\pi})^{\dot{u}_w} (\mu_{z^+})^{\vartheta_w}$$
(34)

for i = 1, ..., N - 1 (note that an agency that was in the *i*-th cohort in period t was in cohort i - 1 in period t - 1) where κ_w , \dot{u}_w , $\kappa_w + \dot{u}_w \in (0,1)$.

After wages are set, the employment agencies in cohort *i* decide on endogenous separation, post vacancies to attract new workers in the next period and supply labour services $l_t^i \varsigma_{i,t}$ to competitive labour markets. Simplifying, we obtain:

$$F(l_t^0, \omega_t) = \sum_{j=0}^{N-1} \beta^j E_t \frac{v_{t+j}}{v_t} \max_{\tilde{v}_{t+j}^j} [(W_{t+j} \mathcal{E}_{t+j}^j - \Gamma_{t,j} \omega_t [1 - \mathcal{F}_{t+j}^j]) \varsigma_{j,t+j} - P_{t+j} \frac{\kappa z_{t+j}^+}{\varphi} (\tilde{v}_t^j)^{\varphi} (1 - \mathcal{F}_{t+j}^j)] l_{t+j}^j + \beta^N E_t \frac{v_{t+N}}{v_t} F(l_{t+N}^0, \widetilde{W}_{t+N})$$

$$(35).$$

For convenience, we omit expectations operator E_t below.

Writing out (35), gives:

$$\begin{split} F(l_t^0, \omega_t) &= \max_{\{v_{t+j}^j\}_{j=0}^{N-1}} \{ \left[(W_t \mathcal{E}_t^0 - \omega_t (1 - \mathcal{F}_t^0))\varsigma_t - P_t \frac{\kappa z_t^+}{\varphi} (\tilde{v}_t^0)^{\varphi} (1 - \mathcal{F}_t^0) \right] l_t^0 \\ &+ \beta E_t \frac{v_{t+1}}{v_t} \left[(W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t (1 - \mathcal{F}_{t+1}^1))\varsigma_{t+1} - P_{t+1} \frac{\kappa z_{t+1}^+}{\varphi} (\tilde{v}_{t+1}^1)^{\varphi} (1 - \mathcal{F}_{t+1}^1) \right] \\ &\times (\chi_t^0 + \rho) [1 - \mathcal{F}_t^0] l_t^0 \\ &+ \beta^2 E_t \frac{v_{t+2}}{v_t} \left[(W_{t+2} \mathcal{E}_{t+2}^2 - \Gamma_{t,2} \omega_t (1 - \mathcal{F}_{t+2}^2))\varsigma_{t+2} - P_{t+2} \frac{\kappa z_{t+2}^+}{\varphi} (\tilde{v}_{t+2}^2)^{\varphi} (1 - \mathcal{F}_{t+2}^2) \right] \\ &\times (\chi_{t+1}^1 + \rho) (\chi_t^0 + \rho) (1 - \mathcal{F}_{t+1}^1) (1 - \mathcal{F}_t^0) l_t^0 \\ &+ \cdots + \\ &+ \beta^N E_t \frac{v_{t+N}}{v_t} F(l_{t+N}^0, \widetilde{W}_{t+N}) \}. \end{split}$$

$$\begin{split} J(\omega_{t}) &= \max_{\{v_{t+j}^{j}\}_{j=0}^{N-1}} \{ (W_{t} \mathcal{E}_{t}^{0} - \omega_{t} (1 - \mathcal{F}_{t}^{0})) \varsigma_{0,t} - P_{t} z_{t}^{+} \frac{\kappa}{\varphi} (\tilde{v}_{t}^{0})^{\varphi} [1 - \mathcal{F}_{t}^{0}] \\ &+ \beta \frac{v_{t+1}}{v_{t}} \bigg[(W_{t+1} \mathcal{E}_{t+1}^{1} - \Gamma_{t,1} \omega_{t} (1 - \mathcal{F}_{t+1}^{1})) \varsigma_{1,t+1} - P_{t+1} z_{t+1}^{+} \frac{\kappa}{\varphi} (\tilde{v}_{t+1}^{1})^{\varphi} (1 - \mathcal{F}_{t+1}^{1}) \bigg] \times \\ &\times (\tilde{v}_{t}^{0} Q_{t}^{1-\iota} + \rho) (1 - \mathcal{F}_{t}^{0}) \\ &+ \beta^{2} \frac{v_{t+2}}{v_{t}} \bigg[(W_{t+2} \mathcal{E}_{t+2}^{2} - \Gamma_{t,2} \omega_{t} (1 - \mathcal{F}_{t+2}^{2})) \varsigma_{2,t+2} - P_{t+2} z_{t+2}^{+} \frac{\kappa}{\varphi} (\tilde{v}_{t+2}^{2})^{\varphi} (1 - \mathcal{F}_{t+2}^{2}) \bigg] \times \\ &\times (\tilde{v}_{t}^{0} Q_{t}^{1-\iota} + \rho) (\tilde{v}_{t+1}^{1} Q_{t+1}^{1-\iota} + \rho) (1 - \mathcal{F}_{t+1}^{1}) [1 - \mathcal{F}_{t}^{0}] \\ &+ \cdots + \\ &+ \beta^{N} \frac{v_{t+N}}{v_{t}} J (\tilde{W}_{t+N}) (\tilde{v}_{t}^{0} Q_{t}^{1-\iota} + \rho) (\tilde{v}_{t+1}^{1} Q_{t}^{1-\iota} + \rho) \cdots (\tilde{v}_{t+N-1}^{N-1} Q_{t+N-1}^{1-\iota} + \rho) \times \\ &\times (1 - \mathcal{F}_{t+N-1}^{N-1}) \cdots (1 - \mathcal{F}_{t}^{0}) \} \end{split}$$
(36).

We derive optimal vacancy posting decisions of employment agencies by differentiating (36) with respect to \tilde{v}_t^0 and multiply the result by $(\tilde{v}_t^0 Q_t^{1-\iota} + \rho)/Q_t^{1-\iota}$, to obtain:

$$\begin{split} 0 &= -P_{t}z_{t}^{+}\kappa(\tilde{v}_{t}^{0})^{\varphi-1}[1-\mathcal{F}_{t}^{0}](\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho)/Q_{t}^{1-\iota} \\ &+\beta\frac{v_{t+1}}{v_{t}}\bigg[(W_{t+1}\mathcal{E}_{t+1}^{1}-\Gamma_{t,1}\omega_{t}[1-\mathcal{F}_{t+1}^{1}])\varsigma_{1,t+1}-P_{t+1}z_{t+1}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^{1})^{\varphi}(1-\mathcal{F}_{t+1}^{1})\bigg] \times \\ &\times(\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho)[1-\mathcal{F}_{t}^{0}] \\ &+\beta^{2}\frac{v_{t+2}}{v_{t}}\bigg[(W_{t+2}\mathcal{E}_{t+2}^{2}-\Gamma_{t,2}\omega_{t}[1-\mathcal{F}_{t+2}^{2}])\varsigma_{2,t+2}-P_{t+2}z_{t+2}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^{2})^{\varphi}(1-\mathcal{F}_{t+2}^{2})\bigg] \times \\ &\times(\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho)(\tilde{v}_{t+1}^{1}Q_{t+1}^{1-\iota}+\rho)[1-\mathcal{F}_{t+1}^{1}][1-\mathcal{F}_{t}^{0}] \\ &+\cdots+ \\ &\beta^{N}\frac{v_{t+N}}{v_{t}}J(\tilde{W}_{t+N})(\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho)(\tilde{v}_{t+1}^{1}Q_{t+1}^{1-\iota}+\rho)\cdots(\tilde{v}_{t+N-1}^{N-1}Q_{t+N-1}^{1-\iota}+\rho) \times \\ &\times[1-\mathcal{F}_{t+N-1}^{N-1}]\cdots[1-\mathcal{F}_{t}^{0}] \\ &=J(\omega_{t})-(W_{t}\mathcal{E}_{t}^{0}-\omega_{t}(1-\mathcal{F}_{t}^{0}))\varsigma_{0,t}+P_{t}z_{t}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t}^{0})^{\varphi}[1-\mathcal{F}_{t}^{0}] \\ &-P_{t}z_{t}^{+}\kappa(\tilde{v}_{t}^{0})^{\varphi-1}[1-\mathcal{F}_{t}^{0}](\tilde{v}_{t}^{0}Q_{t}^{1-\iota}+\rho)/Q_{t}^{1-\iota}. \end{split}$$

Since the latter expression must be zero, we get [some math skipped]:

$$J(\omega_t) = (W_t \mathcal{E}_t^0 - \omega_t (1 - \mathcal{F}_t^0)) \varsigma_{0,t} + P_t z_t^+ \kappa \left[\left(1 - \frac{1}{\varphi} \right) (\tilde{v}_t^0)^{\varphi} + (\tilde{v}_t^0)^{\varphi - 1} \frac{\rho}{Q_t^{1-\iota}} \right] [1 - \mathcal{F}_t^0].$$

Next, we obtain simple expressions for vacancy decisions from their FOCs for optimality. Multiplying FOC for \tilde{v}_{t+1}^1 by

$$(\tilde{v}_{t+1}^1 Q_{t+1}^{1-\iota} + \rho) \frac{1}{Q_{t+1}^{1-\iota'}}$$

substituting out period t + 2 and higher terms using the FOC for \tilde{v}_t^0 and rearranging [some math skipped], we obtain:

$$\frac{P_t z_t^+ \kappa(\tilde{v}_t^0)^{\varphi - 1}}{Q_t^{1 - \iota}} = \beta \frac{v_{t+1}}{v_t} \begin{bmatrix} (W_{t+1} \mathcal{E}_{t+1}^1 - \Gamma_{t,1} \omega_t [1 - \mathcal{F}_{t+1}^1]) \varsigma_{1,t+1} \\ + P_{t+1} z_{t+1}^+ \kappa (1 - \mathcal{F}_{t+1}^1) \left[\left(1 - \frac{1}{\varphi} \right) (\tilde{v}_{t+1}^1)^{\varphi} + (\tilde{v}_{t+1}^1)^{\varphi - 1} \frac{\rho}{Q_{t+1}^{1 - \iota}} \right] \end{bmatrix}.$$

Continuing this way [some math skipped],

$$\frac{P_{t+j}z_{t+j}^{+}\kappa(\tilde{v}_{t+j}^{j})^{\varphi-1}}{Q_{t+j}^{1-\iota}} = \beta \frac{v_{t+j+1}}{v_{t+j}} \left[\begin{pmatrix} (W_{t+j+1}\mathcal{E}_{t+j+1}^{j+1} - \Gamma_{t,j+1}\omega_t [1 - \mathcal{F}_{t+j+1}^{j+1}])\varsigma_{j+1,t+j+1} \\ + P_{t+j+1}z_{t+j+1}^{+}\kappa(1 - \mathcal{F}_{t+j+1}^{j+1}) \\ \begin{pmatrix} (1 - \frac{1}{\varphi})(\tilde{v}_{t+j+1}^{j+1})^{\varphi} \\ + (\tilde{v}_{t+j+1}^{j+1})^{\varphi-1} \frac{\rho}{Q_{t+j+1}^{1-\iota}} \\ \end{pmatrix} \right]$$

for j = 0, 1, ..., N - 2.

Now we consider the FOC for optimality of \tilde{v}_{t+N-1}^{N-1} [some math skipped]:

$$\frac{P_{t+N-1}z_{t+N-1}^{+}\kappa(\tilde{v}_{t+N-1}^{N-1})^{\varphi-1}}{Q_{t+N-1}^{1-t}} = \beta \frac{v_{t+N}}{v_{t+N-1}} \begin{bmatrix} (W_{t+N}\mathcal{E}_{t+N}^{0} - \tilde{W}_{t+N}[1 - \mathcal{F}_{t+N}^{0}])\varsigma_{0,t+N} \\ + P_{t+N}z_{t+N}^{+}\kappa \begin{bmatrix} \left(1 - \frac{1}{\varphi}\right)(\tilde{v}_{t+N}^{0})^{\varphi} \\ +(\tilde{v}_{t+N}^{0})^{\varphi-1}\frac{\rho}{Q_{t+N}^{1-t}} \end{bmatrix} (1 - \mathcal{F}_{t+N}^{0}) \end{bmatrix}$$

The above FOCs apply, over time, to a group of agencies that bargain on date t. We now express the FOCs for a fixed date and different cohorts:

$$\begin{split} P_{t}z_{t}^{+}\kappa(\tilde{v}_{t}^{j})^{\varphi-1}\frac{1}{Q_{t}^{1-\iota}} &= \beta \frac{v_{t+1}}{v_{t}} [\left(W_{t+1}\mathcal{E}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\widetilde{W}_{t-j}(1-\mathcal{F}_{t+1}^{j+1})\right)\varsigma_{j+1,t+1} \\ &+ P_{t+1}z_{t+1}^{+}\kappa(1-\mathcal{F}_{t+1}^{j+1})\left(\left(1-\frac{1}{\varphi}\right)(\tilde{v}_{t+1}^{j+1})^{\varphi} + (\tilde{v}_{t+1}^{j+1})^{\varphi-1}\frac{\rho}{Q_{t+1}^{1-\iota}}\right)] \end{split}$$

for j = 0, ..., N - 2. Scaling by $P_t z_t^+$ yields:

$$\kappa(\tilde{v}_{t}^{j})^{\varphi-1} \frac{1}{Q_{t}^{1-\iota}} = \beta \frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}} [\left(\overline{w}_{t+1} \mathcal{E}_{t+1}^{j+1} - G_{t-j,j+1} w_{t-j} \overline{w}_{t-j} (1 - \mathcal{F}_{t+1}^{j+1})\right) \varsigma_{j+1,t+1} + \kappa (1 - \mathcal{F}_{t+1}^{j+1}) \left(\left(1 - \frac{1}{\varphi}\right) (\tilde{v}_{t+1}^{j+1})^{\varphi} + (\tilde{v}_{t+1}^{j+1})^{\varphi-1} \frac{\rho}{Q_{t+1}^{1-\iota}} \right)]$$

$$(37)$$

for j = 0, ..., N - 2, where

$$G_{t-i,i+1} = \frac{\widetilde{\pi}_{w,t+1}\cdots\widetilde{\pi}_{w,t-i+1}}{\pi_{t+1}\cdots\pi_{t-i+1}} \left(\frac{1}{\mu_{z^+,t-i+1}}\right) \cdots \left(\frac{1}{\mu_{z^+,t+1}}\right), i \ge 0,$$

$$w_t = \frac{\widetilde{w}_t}{w_t}, \overline{w}_t = \frac{w_t}{z_t^{+P_t}}$$
(38),

and

$$G_{t,j} = \begin{cases} \frac{\tilde{\pi}_{w,t+j} \cdots \tilde{\pi}_{w,t+1}}{\pi_{t+j} \cdots \pi_{t+1}} \left(\frac{1}{\mu_{z^+,t+1}} \right) \cdots \left(\frac{1}{\mu_{z^+,t+j}} \right) & j > 0\\ 1 & j = 0 \end{cases}$$
(39).

The scaled vacancy FOC of agencies that are in the last period of their contract is:

$$\kappa(\tilde{v}_{t}^{N-1})^{\varphi-1} \frac{1}{q_{t}^{1-\iota}} = \beta \frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}} [(\bar{w}_{t+1} \mathcal{E}_{t+1}^{0} - w_{t+1} \bar{w}_{t+1} (1 - \mathcal{F}_{t+1}^{0}))\varsigma_{0,t+1} + \kappa (1 - \mathcal{F}_{t+1}^{0}) \left(\left(1 - \frac{1}{\varphi}\right) (\tilde{v}_{t+1}^{0})^{\varphi} + (\tilde{v}_{t+1}^{0})^{\varphi-1} \frac{\rho}{q_{t+1}^{1-\iota}} \right)]$$

$$(40).$$

C3.3 Agency separation decisions

We start by considering the separation decision of a representative agency in the j = 0 cohort which renegotiates the wage in the current period. After that, we consider j > 0.

Separation decisions of agencies renegotiating their wages in current period

We start by considering the impact of \bar{a}_t^0 on agency and worker's surplus respectively. The aggregate surplus across all the l_t^0 workers in the representative agency is given by (24). The object \mathcal{F}_t^0 is a function of \bar{a}_t^0 as indicated in (6). We denote its derivative by

$$\mathcal{F}_t^{j\prime} := \frac{d\mathcal{F}_t^j}{d\bar{a}_t^j} \tag{41}$$

for . = 0, ..., -1. Where convenient, in this subsection we include expressions that apply to the representative agency in cohort > 0 as well as to those in cohort = 0. According to (11), \bar{a}_t^0 affects V_t^0 via its impact on hours worked ς_j . Hours worked is a function of \bar{a}_t^0 because \mathcal{G}_t^0 is (see (12), (11) and (19)). These observations about V_t^0 also apply to V_t^j for > 0. Thus, differentiating (19), yields

$$V_t^{j\prime} := \frac{d}{d\bar{a}_t^j} V_t^j = \left[\Gamma_{t-j,j} \widetilde{W}_{t-j} \frac{1-\tau^y}{1+\tau^w} - A_L \frac{\zeta_t \varsigma_{j,t}^{\sigma_L}}{v_t} \right] \varsigma_{j,t}^{\prime}$$
(42)

where

$$\varsigma_{j,t}' := \frac{d\varsigma_{j,t}}{d\bar{a}_t^j} = \frac{1}{\sigma_L} (\varsigma_{j,t})^{1 - \sigma_L} \frac{W_t v_t}{\zeta_t A_L} \frac{1 - \tau^y}{1 + \tau^w} \mathcal{G}_t^{j\prime}$$
(43)

and

$$\mathcal{G}_t^{j\prime} := \frac{d\mathcal{G}_t^j}{d\bar{a}_t^j} \tag{44}.$$

The counterpart to (43) in terms of scaled variables is:

$$\varsigma_{j,t}' := \frac{1}{\sigma_L} (\varsigma_{j,t})^{1 - \sigma_L} \frac{\overline{w}_t w_t \psi_{z^+,t}}{\zeta_t A_L} \frac{1 - \tau^{\gamma}}{1 + \tau^w} \mathcal{G}_t^{j\prime}$$
(45).

The value of being unemployed U_t is not a function of \bar{a}_t^0 chosen by the representative agency, because U_t is determined by economy-wide aggregate variables, such as the job finding rate (see (20)).

According to (18), the agency surplus per worker in l_t^0 is given by $J(\omega_t)$, and it has the following representation:

$$J(\omega_t) = \max_{\bar{a}_t^0} \tilde{J}(\omega_t; \bar{a}_t^0)(1 - \mathcal{F}_t^0)$$

where $\tilde{J}(\omega_t; \bar{a}_t^0)$ is given by (25) and

$$J_{t+1}^{j+1}(\omega_{t}) = \max_{\{\tilde{a}_{t+i}^{i}, \tilde{v}_{t+i}^{i}\}_{t=j}^{N-1}} \{ \left[(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^{j+1})^{\varphi} \right] \\ \times (1 - \mathcal{F}_{t+1}^{j+1}) \\ \beta \frac{v_{t+2}}{v_{t+1}} \left[(W_{t+2}\mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2}\omega_{t-j})\varsigma_{j+2,t+2} - P_{t+2}z_{t+2}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^{j+2})^{\varphi} \right] \\ \times (1 - \mathcal{F}_{t+2}^{j+2})(\chi_{t+1}^{j+1} + \rho)(1 - \mathcal{F}_{t+1}^{j+1}) \\ + \cdots + \\ + \beta^{N-j-1}\frac{v_{t+N-j}}{v_{t+1}}J(\tilde{W}_{t+N-j})(\chi_{t+N-j-1}^{N-1} + \rho)(1 - \mathcal{F}_{t+N-j-1}^{N-1})\cdots(\chi_{t+1}^{j+1} + \rho)(1 - \mathcal{F}_{t+1}^{j+1}) \}$$
(46)
for $j = 0$.

In (25) and (46), it is understood that χ_{t+j}^{j} , \tilde{v}_{t+j}^{j} are connected by (14). Thus, the surplus of the representative agency with workforce l_{t}^{0} expressed as a function of an arbitrary value of \bar{a}_{t}^{0} is given by (26). Differentiation of \tilde{J} with respect to \bar{a}_{t}^{j} only

needs to be concerned with the impact of \bar{a}_t^j on \mathcal{G}_t^j and $\varsigma_{j,t}$. Generalising (25) to cohort *j*, produces:

$$\tilde{J}(\omega_{t-j}; \bar{a}_t^j) = \max_{\tilde{v}_t^j} \Big\{ (W_t \mathcal{G}_t^j - \Gamma_{t-j,j} \omega_{t-j}) \varsigma_{j,t} - P_t z_t^+ \frac{\kappa}{\varphi} (\tilde{v}_t^j)^\varphi + \beta \frac{v_{t+1}}{v_t} (\chi_t^j + \rho) J_{t+1}^{j+1} (\omega_{t-j}) \Big\}.$$

Then,

$$\tilde{J}_{\bar{a}^{j}}(\omega_{t-j};\bar{a}_{t}^{j}):=\frac{d\tilde{J}(\omega_{t-j};\bar{a}_{t}^{j})}{d\bar{a}_{t}^{j}}=(W_{t}\mathcal{G}_{t}^{j}-\Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t}'+W_{t}\mathcal{G}_{t}^{j'}\varsigma_{j,t}$$
(47)

where $\varsigma'_{i,t}$ and $\mathcal{G}^{j'}_t$ are defined in (43) and (44) respectively.

We now evaluate $\mathcal{F}_t^{j'}$ and $\mathcal{G}_t^{j'}$ for $j \ge 0$. It is assumed that productivity a is drawn from a log-normal distribution having the properties: E(a) = 1 and $V(\log(a)) = \sigma_a^2$. This assumption simplifies the analysis because analytic expressions are available for such objects as $\mathcal{F}_t^{j'}$, $\mathcal{G}_t^{j'}$. Although these expressions are readily available in the literature (see, e.g. BGG), we derive them for completeness. It is easily verified that \mathcal{F} has the following representation:¹¹

$$\mathcal{F}(\bar{a}^{j};\sigma_{a}) = \frac{1}{\sigma_{a}\sqrt{2\pi}} \int_{-\infty}^{\log(\bar{a}^{j})} e^{x} e^{\frac{-\left(x+\frac{1}{2}\sigma_{a}^{2}\right)^{2}}{2\sigma_{a}^{2}}} dx$$

where $x = \log a$. Combining the exponential terms, gives:

$$\mathcal{F}(\bar{a}^{j};\sigma_{a}) = \frac{1}{\sigma_{a}\sqrt{2\pi}} \int_{-\infty}^{\log(\bar{a}^{j})} \exp\left[\frac{-\left(x-\frac{1}{2}\sigma_{a}^{2}\right)^{2}}{2\sigma_{a}^{2}}\right] dx.$$

Making the change of variables,

$$v := \frac{x - \frac{1}{2}\sigma_a^2}{\sigma_a}$$

so that

$$dv = \frac{1}{\sigma_a} dx$$

and substituting into the expression for \mathcal{F} ,

$$\mathcal{F}(\bar{a}^{j};\sigma_{a}) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\log(\bar{a}^{j}) + \frac{1}{2}\sigma_{a}^{2}} \exp(\frac{-\nu^{2}}{2}) d\nu.$$

This is just the standard normal cumulative distribution evaluated at $(\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2)/\sigma_a$. Differentiating \mathcal{F} , we obtain the following expression for (41):

$$\mathcal{F}_t^{j\prime} = \frac{1}{\bar{a}^j \sigma_a \sqrt{2\pi}} \exp\left(-\frac{\left(\log(\bar{a}^j) + \frac{1}{2}\sigma_a^2\right)^2}{2\sigma_a^2}\right)$$
(48).

The object on the right of the equality is just the normal density with variance σ_a^2 and mean $-\sigma_a^2/2$, evaluated at log(\bar{a}^j) and divided by \bar{a}^j . From (13), we obtain:

¹¹ E(a) = 1 is when $E(\log(a)) = -\sigma_a^2/2$.

$$\mathcal{E}_t^{j\prime} = -\bar{a}_t^j \mathcal{F}_t^{j\prime} \tag{49}.$$

Differentiating (44), yields:

$$\mathcal{G}_{t}^{j\prime} = \frac{\mathcal{E}_{t}^{j\prime}(1-\mathcal{F}_{t}^{j})+\mathcal{E}_{t}^{j}\mathcal{F}_{t}^{j\prime}}{[1-\mathcal{F}_{t}^{j}]^{2}}$$
(50).

The surplus criterion governing the choice of \bar{a}_t^0 is (27). FOC for an interior optimum is given by (28), which is reproduced here for convenience:

$$s_w V_t^{0\prime} + s_e \tilde{J}_{\bar{a}^0} \big(\widetilde{W}_t; \bar{a}_t^0 \big) = \big[s_w (V_t^0 - U_t) + s_e \tilde{J} \big(\widetilde{W}_t; \bar{a}_t^0 \big) \big] \frac{\mathcal{F}_t^{0\prime}}{1 - \mathcal{F}_t^0}$$

where the fact is used that the wage paid to workers in the bargaining period is denoted by \widetilde{W}_t . After substituting from (42) and (47), we obtain:

$$s_{w}\left(\widetilde{W}_{t}\frac{1-\tau_{y}}{1+\tau^{w}}-A_{L}\frac{\zeta_{t}\varsigma_{0,t}^{o_{L}}}{\upsilon_{t}}\right)\varsigma_{0,t}'+s_{e}\left[\left(W_{t}\mathcal{G}_{t}^{0}-\widetilde{W}_{t}\right)\varsigma_{0,t}'+W_{t}\mathcal{G}_{t}^{0\prime}\varsigma_{0,t}\right]=\left[s_{w}(V_{t}^{0}-U_{t})+s_{e}\widetilde{J}\left(\widetilde{W}_{t};\overline{a}_{t}^{0}\right)\right]\frac{\mathcal{F}_{t}^{0\prime}}{1-\mathcal{F}_{t}^{0}}$$
(51).

In scaled terms and dividing by $P_t z_t^+$ yields [some math skipped]:

$$s_{w}\left(w_{t}\overline{w}_{t}\frac{1-\tau^{y}}{1+\tau^{w}}-A_{L}\frac{\zeta_{t}\varsigma_{0,t}^{\sigma_{L}}}{\psi_{z^{+},t}}\right)\varsigma_{0,t}'+s_{e}\overline{w}_{t}\left[(\mathcal{G}_{t}^{0}-w_{t})\varsigma_{0,t}'+\mathcal{G}_{t}^{0'}\varsigma_{0,t}\right]=\left[s_{w}\left(V_{z^{+},t}^{0}-U_{z^{+},t}\right)\right]\frac{\mathcal{F}_{t}^{0'}}{1-\mathcal{F}_{t}^{0}}$$
(52).

Separation decisions of agencies having renegotiated in previous periods

We now turn to \bar{a}_t^j decision for j = 1, ..., N - 1. The representative agency that selects \bar{a}_t^j is a member of the cohort of agencies that bargained *j* periods in the past. We denote the present discounted value of profits of the representative agency in cohort *j* by $F_t^j(\omega_{t-j})$:

$$\begin{aligned} \frac{F_{t}^{j}(l_{t}^{j},\omega_{t-j})}{l_{t}^{j}} &:= J_{t}^{j}(\omega_{t-j}) = \max_{\{\bar{a}_{t+i}^{j+i},\tilde{v}_{t+i}^{j+i}\}_{i=0}^{i} \in \{\left[(W_{t}\mathcal{G}_{t}^{j} - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_{t}z_{t}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t}^{j})^{\varphi}\right] \\ &\times (1 - \mathcal{F}_{t}^{j}) \\ &+ \beta \frac{v_{t+1}}{v_{t}} \Big[(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^{j+1})^{\varphi}\Big] \\ &\times (1 - \mathcal{F}_{t+1}^{j+1})(\chi_{t}^{j} + \rho)(1 - \mathcal{F}_{t}^{j}) \\ &+ \cdots + \\ &+ \beta^{N-j} \frac{v_{t+N-j}}{v_{t}} J(\tilde{W}_{t+N-j})(\chi_{t+N-1-j}^{N-1} + \rho)(1 - \mathcal{F}_{t+N-j-1}^{N-1}) \cdots \\ &\times (\chi_{t}^{j} + \rho)(1 - \mathcal{F}_{t}^{j}) \}. \end{aligned}$$

Here, we use that $F_t^j(l_t^j, \omega_{t-j})$ is proportional to l_t^j as in the case of j = 0 considered in (18). In particular, $J_t^j(\omega_{t-j})$ is not a function of l_t^j and corresponds to the object in (46), with the time index t replaced by t - j. The term $J_t^j(\omega_{t-j})$ can be written as

$$J_t^j(\omega_{t-j}) = \tilde{J}_t^j(\omega_{t-j}; \bar{a}_t^j)(1 - \mathcal{F}_t^j)$$

where

$$\tilde{J}(\omega_{t-j}; \bar{a}_t^j) = (W_t \mathcal{G}_t^j - \Gamma_{t-j,j} \omega_{t-j}) \varsigma_{j,t} - P_t z_t^+ \frac{\kappa}{\varphi} (\tilde{v}_t^j)^\varphi + \beta \frac{v_{t+1}}{v_t} J_{t+1}^{j+1}(\omega_{t-j}) (\chi_t^j + \rho)$$

from a generalisation of (25) for j = 1, ..., N - 1.

In this way, we obtain an expression for agency surplus for agencies that have not negotiated for j periods, which is symmetric to (26):

$$F_t^{j}(\omega_{t-j}) = \tilde{f}_t^{j}(\omega_{t-j}; \bar{a}_t^{j})(1 - \mathcal{F}_t^{j})l_t^{j}$$
(53).

The expression for total surplus is the analogue of (27):

$$\left[s_{w}(V_{t}^{j} - U_{t}) + s_{e}\tilde{J}_{t}^{j}(\omega_{t-j};\bar{a}_{t}^{j})\right](1 - \mathcal{F}_{t}^{j})l_{t}^{j}$$
(54).

Differentiating, we obtain:

$$s_{w}V_{t}^{j\prime} + s_{e}\tilde{J}_{\bar{a}^{j}}(\omega_{t-j};\bar{a}_{t}^{j}) = \left[s_{w}(V_{t}^{j} - U_{t}) + s_{e}\tilde{J}_{t}^{j}(\omega_{t-j};\bar{a}_{t}^{j})\right]\frac{\mathcal{F}_{t}^{j\prime}}{1 - \mathcal{F}_{t}^{j}}$$
(55),

which corresponds to (28). Here, $\tilde{J}_{\bar{a}^j}(\omega_{t-1}; \bar{a}_t^j)$ is the analogue of (47), with index 0 being replaced by *j*. After substituting from the analogues for cohort *j* of (42) and (47), we get:

$$s_{w}\left(\Gamma_{t-j,j}\widetilde{W}_{t-j}\frac{1-\tau^{y}}{1+\tau^{w}}-A_{L}\frac{\zeta_{t}\varsigma_{j,t}^{o_{L}}}{\upsilon_{t}}\right)\varsigma_{j,t}'+s_{e}\left[\left(W_{t}\mathcal{G}_{t}^{j}-\Gamma_{t-j,j}\widetilde{W}_{t-j}\right)\varsigma_{j,t}'+W_{t}\mathcal{G}_{t}^{j'}\varsigma_{j,t}\right]$$
$$=\left[s_{w}(V_{t}^{j}-U_{t})+s_{e}\widetilde{J}(\widetilde{W}_{t-j};\overline{a}_{t}^{j})\right]\frac{\mathcal{F}_{t}^{j'}}{1-\mathcal{F}_{t}^{j}}.$$

Scaling analogously to (52) and plugging in $\widetilde{W}_{t-j} = w_{t-j}\overline{w}_{t-j}P_{t-j}z_{t-j}^+$ and $\overline{w}_t z_t^+ P_t = W_t$, yields:

$$s_{w}\left(G_{t-j,j}w_{t-j}\overline{w}_{t-j}\frac{1-\tau^{y}}{1+\tau^{w}}-A_{L}\frac{\zeta_{t}\varsigma_{j,t}^{\sigma_{L}}}{\psi_{z^{+},t}}\right)\varsigma_{j,t}'+s_{e}\left[\left(\overline{w}_{t}G_{t}^{j}-G_{t-j,j}\overline{w}_{t-j}w_{t-j}\right)\varsigma_{j,t}'+\overline{w}_{t}G_{t}^{j'}\varsigma_{j,t}\right]$$

$$=\left[s_{w}(V_{z^{+},t}^{j}-U_{z^{+},t})+s_{e}\tilde{J}_{z^{+},t}^{j}\right]\frac{\mathcal{F}_{t}^{j'}}{1-\mathcal{F}_{t}^{j}}$$
(56).

Finally, we need an explicit expression for $\tilde{J}(\tilde{W}_t; \bar{a}_t^j)$ or rather its scaled equivalent $\tilde{J}_{z^+,t}^j$. For this, we use (46) to write out $J_{t+1}^{j+1}(\omega_{t-j})$ for j = 1, ..., N, and plug into (25):

$$\tilde{J}_{t}^{j}(\omega_{t-j};\bar{a}_{t}^{j}) = (W_{t}\mathcal{G}_{t}^{j} - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_{t}z_{t}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t}^{j})^{\varphi} + \beta\frac{v_{t+1}}{v_{t}}J_{t+1}^{j+1}(\omega_{t-j})(\chi_{t}^{j} + \rho)$$

Using (46), yields:

$$\begin{split} \tilde{J}_{t}^{j}(\omega_{t-j};\bar{a}_{t}^{j}) &= (W_{t}\mathcal{G}_{t}^{j} - \Gamma_{t-j,j}\omega_{t-j})\varsigma_{j,t} - P_{t}z_{t}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t}^{j})^{\varphi} + \beta\frac{v_{t+1}}{v_{t}}(\chi_{t}^{j} + \rho) \{ \\ &\left[(W_{t+1}\mathcal{G}_{t+1}^{j+1} - \Gamma_{t-j,j+1}\omega_{t-j})\varsigma_{j+1,t+1} - P_{t+1}z_{t+1}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+1}^{j+1})^{\varphi} \right] (1 - \mathcal{F}_{t+1}^{j+1}) \\ &+ \beta\frac{v_{t+2}}{v_{t+1}} \Big[(W_{t+2}\mathcal{G}_{t+2}^{j+2} - \Gamma_{t-j,j+2}\omega_{t-j})\varsigma_{j+2,t+2} - P_{t+2}z_{t+2}^{+}\frac{\kappa}{\varphi}(\tilde{v}_{t+2}^{j+2})^{\varphi} \Big] \\ &\times (1 - \mathcal{F}_{t+2}^{j+2})(\chi_{t+1}^{j+1} + \rho)(1 - \mathcal{F}_{t+1}^{j+1}) \\ &+ \cdots + \\ &+ \beta^{N-j-1}\frac{v_{t+N-j}}{v_{t+1}} J \big(\tilde{W}_{t+N-j} \big) (\chi_{t+N-j-1}^{N-1} + \rho)(1 - \mathcal{F}_{t+N-j-1}^{N-1}) \cdots \\ &\times (\chi_{t+1}^{j+1} + \rho)(1 - \mathcal{F}_{t+1}^{j+1}) \} \end{split}$$

for j = 0, ..., N - 1. Plugging in for $\omega_{t-j} = \widetilde{W}_{t-j} = w_{t-j}\overline{w}_{t-j}P_{t-j}z_{t-j}^+$, scaling and rearranging [some math skipped], give:

$$\begin{split} \tilde{J}_{z^{+},t}^{j}(\widetilde{W}_{t-j}; \bar{a}_{t}^{j}) &:= \frac{\tilde{J}^{j}(\widetilde{W}_{t}; \bar{a}_{t}^{j})}{P_{t} z_{t}^{+}} = (\overline{w}_{t} \mathcal{G}_{t}^{j} - \mathcal{G}_{t-j,j} w_{t-j} \overline{w}_{t-j}) \varsigma_{j,t} - \frac{\kappa}{\varphi} (\widetilde{v}_{t}^{j})^{\varphi} \\ &+ \beta \frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}} (\chi_{t}^{j} + \rho) \{ \\ \left[(\overline{w}_{t+1} \mathcal{G}_{t+1}^{j+1} - \mathcal{G}_{t-j,j+1} w_{t-j} \overline{w}_{t-j}) \varsigma_{j+1,t+1} - \frac{\kappa}{\varphi} (\widetilde{v}_{t+1}^{j+1})^{\varphi} \right] (1 - \mathcal{F}_{t+1}^{j+1}) \\ &+ \beta \frac{\psi_{z^{+},t+2}}{\psi_{z^{+},t+1}} \left[(\overline{w}_{t+2} \mathcal{G}_{t+2}^{j+2} - \mathcal{G}_{t-j,j+2} w_{t-j} \overline{w}_{t-j}) \varsigma_{j+2,t+2} - \frac{\kappa}{\varphi} (\widetilde{v}_{t+2}^{j+2})^{\varphi} \right] \\ &\times (1 - \mathcal{F}_{t+2}^{j+2}) (\chi_{t+1}^{j+1} + \rho) (1 - \mathcal{F}_{t+1}^{j+1}) \\ &+ \cdots + \\ &+ \beta^{N-j-1} \frac{\psi_{z^{+},t+N-j}}{\psi_{z^{+},t+1}} J_{z^{+},t+N-j} (\chi_{t+N-j-1}^{N-1} + \rho) (1 - \mathcal{F}_{t+N-j-1}^{N-1}) \cdots \\ &\times (\chi_{t+1}^{j+1} + \rho) (1 - \mathcal{F}_{t+1}^{j+1}) \} \end{split}$$
(57).

C3.4 Bargaining problem

The FOC associated with the Nash bargaining problem, after division by $z_t^+ P_t$, is:

$$\eta_t V_{w,t} J_{z^+,t} + (1 - \eta_t) [V_{z^+,t}^0 - U_{z^+,t}] J_{w,t} = 0$$
(58)

The following is an expression for J_t , evaluated at $\omega_t = \widetilde{W}_t$, in terms of scaled variables

$$J_{z^{+},t} = \sum_{j=0}^{N-1} \beta^{j} \frac{\psi_{z^{+},t+j}}{\psi_{z^{+},t}} \Big[(\overline{w}_{t+j} \mathcal{G}_{t}^{j} - \mathcal{G}_{t,j} w_{t} \overline{w}_{t}) \varsigma_{j,t+j} - \frac{\kappa}{\varphi} (\tilde{v}_{t+j}^{j})^{\varphi} \Big] \Omega_{t+j}^{j} \\ + \beta^{N} \frac{\psi_{z^{+},t+N}}{\psi_{z^{+},t}} J_{z^{+},t+N} \frac{\Omega_{t+N}^{N}}{1 - \mathcal{F}_{t+N}^{0}}$$
(59).

The derivative of J with respect to ω_t , i.e. the marginal surplus of the employment agency with respect to the negotiated wage, is also required. By the envelope

condition, we can ignore the impact of a change in ω_t on endogenous separations and vacancy decisions, and be concerned only with the direct impact of ω_t on J. Taking the derivative of (36), yields:

$$\begin{split} J_{w,t} &= -(1 - \mathcal{F}_{t}^{0})\varsigma_{0,t} \\ &-\beta \frac{v_{t+1}}{v_{t}} \Gamma_{t,1}\varsigma_{1,t+1}(\chi_{t}^{0} + \rho)(1 - \mathcal{F}_{t+1}^{1})(1 - \mathcal{F}_{t}^{0}) \\ &-\beta^{2} \frac{v_{t+2}}{v_{t}} \Gamma_{t,2}\varsigma_{2,t+2}(\chi_{t}^{0} + \rho)(\chi_{t+1}^{1} + \rho)(1 - \mathcal{F}_{t+2}^{2})(1 - \mathcal{F}_{t+1}^{1})(1 - \mathcal{F}_{t}^{0}) \\ &- \cdots - \beta^{N-1} \frac{v_{t+N-1}}{v_{t}} \Gamma_{t,N-1}\varsigma_{N-1,t+N-1}(\chi_{t}^{0} + \rho)(\chi_{t+1}^{1} + \rho) \cdots (\chi_{t+1}^{N-2} + \rho) \times \\ &\times (1 - \mathcal{F}_{t+N-1}^{N-1}) \cdots (1 - \mathcal{F}_{t}^{0}). \end{split}$$

Let

$$\Omega_{t+j}^{j} = \begin{cases} (1 - \mathcal{F}_{t+j}^{j}) \prod_{l=0}^{j-1} (\chi_{t+l}^{l} + \rho)(1 - \mathcal{F}_{t+l}^{l}) & j > 0\\ 1 - \mathcal{F}_{t}^{0} & j = 0 \end{cases}$$
(60).

It is convenient to express it in a recursive form:

$$\Omega_{t}^{0} = 1 - \mathcal{F}_{t}^{0}, \Omega_{t+1}^{1} = (1 - \mathcal{F}_{t+1}^{1})(\chi_{t}^{0} + \rho) \underbrace{(1 - \mathcal{F}_{t}^{0})}_{\Omega_{t+1}^{1}},$$
$$\Omega_{t+2}^{2} = (1 - \mathcal{F}_{t+2}^{2})(\chi_{t+1}^{1} + \rho) \underbrace{(\chi_{t}^{0} + \rho)(1 - \mathcal{F}_{t}^{0})(1 - \mathcal{F}_{t+1}^{1})}_{\text{so that}},$$
so that

$$\Omega_{t+j}^{j} = (1 - \mathcal{F}_{t+j}^{j})(\chi_{t+j-1}^{j-1} + \rho)\Omega_{t+j-1}^{j-1}$$

for j = 1, 2, ... It is convenient to define these objects at date t as a function of variables dated t and earlier for the purpose of implementing these equations in Dynare:

$$\Omega_{t}^{0} = 1 - \mathcal{F}_{t}^{0}, \Omega_{t}^{1} = (1 - \mathcal{F}_{t}^{1})(\chi_{t-1}^{0} + \rho) \underbrace{\overbrace{(1 - \mathcal{F}_{t-1}^{0})}^{\Omega_{t-1}^{0}}}_{\Omega_{t}^{1}},$$

$$\Omega_{t}^{2} = (1 - \mathcal{F}_{t}^{2})(\chi_{t-1}^{1} + \rho) \underbrace{(\chi_{t-2}^{0} + \rho)(1 - \mathcal{F}_{t-2}^{0})(1 - \mathcal{F}_{t-1}^{1})}_{\Omega_{t-1}^{0}},$$

so that

so that

$$\Omega_t^j = (1 - \mathcal{F}_t^j)(\chi_{t-1}^{j-1} + \rho)\Omega_{t-1}^{j-1}.$$

Then, in terms of scaled variables, we get:

$$J_{w,t} = -\sum_{j=0}^{N-1} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} G_{t,j} \Omega^j_{t+j} \varsigma_{j,t+j}$$
(61).

Scaling V_t^i by $P_t z_t^+$, gives:

$$V_{z^+,t}^i = G_{t-i,i} w_{t-i} \overline{w}_{t-i} \varsigma_{i,t} \frac{1-\tau^y}{1+\tau^w} - \zeta_t^h A_L \frac{\varsigma_{i,t}^{1+\sigma_L}}{(1+\sigma_L)\psi_{z^+,t}}$$

$$+\beta E_t \frac{\psi_{z^+,t^{+1}}}{\psi_{z^+,t}} \Big[\rho (1 - \mathcal{F}_{t+1}^{i+1}) V_{z^+,t^{+1}}^{i+1} + (1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}) U_{z^+,t^{+1}} \Big]$$
(62)

for i = 0, 1, ..., N - 1, where

$$\frac{V_t^i}{P_t z_t^+} = V_{z^+,t}^i, U_{z^+,t+1} = \frac{U_{t+1}}{P_{t+1} z_{t+1}^+}.$$

In the analysis of the Nash bargaining problem, we must have the derivative of V_t^0 with respect to the wage rate. To define this derivative, it is useful to have:

$$\mathcal{M}_{t+j} = (1 - \mathcal{F}_t^0) \cdots (1 - \mathcal{F}_{t+j}^j) \tag{63}$$

for j = 0, ..., N - 1. Then, the derivative of V^0 , denoted as $V_w^0(\omega_t)$, is:

$$V_{w}^{0}(\omega_{t}) = E_{t} \sum_{j=0}^{N-1} (\beta \rho)^{j} \mathcal{M}_{t+j} \zeta_{j,t+j} \frac{1-\tau^{y}}{1+\tau^{w}} \Gamma_{t,j} \frac{v_{t+j}}{v_{t}}$$
$$= E_{t} \sum_{j=0}^{N-1} (\beta \rho)^{j} \mathcal{M}_{t+j} \zeta_{j,t+j} \frac{1-\tau^{y}}{1+\tau^{w}} G_{t,j} \frac{\psi_{z^{+},t+j}}{\psi_{z^{+},t}}$$
(64).

Note that ω_t has no impact on the intensity of labor effort. This is determined by (33), independent of the wage rate paid to workers.

Scaling (20), gives:

$$U_{z^{+},t} = b^{u}(1-\tau^{y}) + \beta E_{t} \frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}} [f_{t}V_{z^{+},t+1}^{x} + (1-f_{t})U_{z^{+},t+1}]$$
(65).

This value function applies to any unemployed worker, either they got that way because they were unemployed in the previous period and did not find a job, or they arrived into unemployment because of exogenous or endogenous separation.

C3.5 Resource constraint in full model

It is assumed that the posting of vacancies uses the homogeneous domestic good. We leave the production technology equation

$$y_t = (p_t)^{\frac{\lambda_d}{\lambda_d - 1}} \left[\varepsilon_t \left(\frac{1}{\mu_{\Psi,t}} \frac{1}{\mu_{Z^*,t}} k_t \right)^{\alpha} \left(w_t^{-\frac{\lambda_W}{1 - \lambda_W}} h_t \right)^{1 - \alpha} - \phi \right]$$
(66)

unchanged and alter the resource constraint:

$$y_{t} - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_{t}^{j})^{2} (1 - \mathcal{F}_{t}^{j}) l_{t}^{j} = g_{t} + c_{t}^{d} + i_{t}^{d} + (R_{t}^{x})^{\eta_{x}} [\omega_{x} (p_{t}^{m,x})^{1-\eta_{x}} + (1 - \omega_{x})]^{\frac{\eta_{x}}{1-\eta_{x}}} (1 - \omega_{x}) (p_{t}^{x})^{-\eta_{f}} y_{t}^{*}$$
(67).

Measured GDP is y_t , adjusted for both recruitment (hiring) costs and capital utilisation costs:

$$gdp_t = y_t - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_t^j)^2 (1 - \mathcal{F}_t^j) l_t^j - (p_t^i)^{\eta_i} \left(a(u_t) \frac{\bar{k}_t}{\mu_{\psi,t}\mu_{z^+,t}} \right) (1 - \omega_i).$$

C3.6 Final equilibrium conditions

Total job matches must also satisfy the following matching function:¹²

$$m_t = \sigma_m (1 - L_t)^{\sigma} v_t^{1 - \sigma} \tag{68}$$

where

$$L_t = \sum_{j=0}^{N-1} (1 - \mathcal{F}_t^j) l_t^j$$
(69)

and σ_m is the productivity of the matching technology.

In this environment, there is a distinction between the effective hours and the measured hours. The effective hours are the hours of each person, adjusted by their productivity a. As stated above, the average productivity of a worker working in cohort j (i.e. one who has survived the endogenous productivity cut) is $\mathcal{E}_t^j/(1 - \mathcal{F}_t^j)$. The number of workers who survive productivity cut in cohort j is $(1 - \mathcal{F}_t^j)l_t^j$, so that our measure of total effective hours is:

$$H_{t} = \sum_{j=0}^{N-1} \varsigma_{j,t} \mathcal{E}_{t}^{j} l_{t}^{j}$$
(70).

In contrast, total measured hours are expressed as follows:

$$H_t^{meas} = \sum_{j=0}^{N-1} \zeta_{j,t} (1 - \mathcal{F}_t^j) l_t^j.$$

The job finding rate is
$$f_t = \frac{m_t}{1 - L_t}$$
(71).

The probability of filling a vacancy is

$$Q_t = \frac{m_t}{v_t} \tag{72}.$$

Total vacancies v_t are related to vacancies posted by individual cohorts as follows:

$$v_t = \frac{1}{Q_t^{\iota}} \sum_{j=0}^{N-1} \tilde{v}_t^j (1 - \mathcal{F}_t^j) l_t^j.$$

Note however, that this equation does not add a constraint to the model equilibrium, because it can be derived from equilibrium equations (72), (22) and (14).

C3.7 Characterisation of bargaining set

Implicitly, it was assumed that the scaled wage

$$w_t^i = \frac{W_t^i}{z_t^+ P_t},$$

paid by an employment agency which has renegotiated most recently *i* periods in the past, is always inside the bargaining set $[\underline{w}_t^i, \overline{w}_t^i]$, i = 0, 1, ..., N - 1. Here, \overline{w}_t^i has

¹² In this paper, the Cobb–Douglas specification of the matching function is used. This is not the case in dHRW (2000) where they use $m_t = \frac{(1-L_t)v_t}{((1-L_t)^l + v_t^l)^l}$ (with parameter l = 1.27 calibrated for the US data). For comparison, see Subsection

the property that if $w_t^i > \overline{w}_t^i$, then the agency prefers not to employ the worker; in turn, \underline{w}_t^i has the property that if $w_t^i < \underline{w}_t^i$, then the worker prefers to be unemployed. We now describe our strategy for computing w_t^i and \overline{w}_t^i .

The lower bound \underline{w}_t^i sets to zero the surplus of a worker $(1 - \mathcal{F}_t^i)(V_{z^+,t}^i - U_{z^+,t})$ of an agency in the cohort. By (62),

$$\begin{aligned} U_{z^{+},t} &= \underline{w}_{t}^{i} \varsigma_{i,t} \frac{1 - \tau^{y}}{1 + \tau^{w}} - \zeta_{t}^{h} A_{L} \frac{\varsigma_{i,t}^{1 + \sigma_{L}}}{(1 + \sigma_{L})\psi_{z^{+},t}} \\ &+ \beta E_{t} \frac{\psi_{z^{+},t+1}}{\psi_{z^{+},t}} \Big[\rho (1 - \mathcal{F}_{t+1}^{i+1}) V_{z^{+},t+1}^{i+1} + (1 - \rho + \rho \mathcal{F}_{t+1}^{i+1}) U_{z^{+},t+1} \Big] \end{aligned}$$

for i = 0, ..., N - 1. In steady state, this is

$$\underline{w}_{t}^{i} = \frac{U_{z^{+}} + \zeta^{h} A_{L} \frac{\varsigma_{i}^{1+\sigma_{L}}}{(1+\sigma_{L})\psi_{z^{+}}} - \beta[\rho(1-\mathcal{F}^{i+1})V_{z^{+}}^{i+1} + (1-\rho+\rho\mathcal{F}^{i+1})U_{z^{+}}]}{\varsigma_{i} \frac{1-\tau^{y}}{1+\tau^{w}}}$$

where a variable without time subscript denotes its steady state value.

We now consider the upper bound \overline{w}_t^i , which sets surplus $J_{z^+,t}$ of an agency in cohort *i* to zero, i = 0, ..., N - 1. From (59), we obtain:

$$\begin{split} 0 &= \sum_{j=0}^{N-1-i} \beta^j \frac{\psi_{z^+,t+j}}{\psi_{z^+,t}} \bigg[\bigg(\overline{w}_{t+j} \frac{\varepsilon_{t+j}^j}{1-\mathcal{F}_{t+j}^j} - G_{t,j} \overline{w}_t^i \bigg) \varsigma_{j,t+j} - \frac{\kappa}{\varphi} (\tilde{v}_{t+j}^j)^{\varphi} \bigg] \Omega_{t+j}^j \\ &+ \beta^{N-i} \frac{\psi_{z^+,t+N-i}}{\psi_{z^+,t}} J_{z^+,t+N-i} \frac{\Omega_{t+N-i}^{N-i}}{1-\mathcal{F}_{t+N-i}^0} \end{split}$$

for i = 0, ..., N - 1. In steady state,

$$0 = \sum_{j=0}^{N-1-i} \beta^{j} \left[\left(\overline{w} \frac{\varepsilon^{j}}{1-\mathcal{F}^{j}} - G_{j} \overline{w}^{i} \right) \varsigma_{j} - \frac{\kappa}{\varphi} (\tilde{v}^{j})^{\varphi} \right] \Omega^{j} + \beta^{N-i} J_{z^{+}} \frac{\Omega^{N-i}}{1-\mathcal{F}^{0}}.$$

For the dynamic economy, the additional unknowns are 2N variables composed of \underline{w}_t^i and \overline{w}_t^i for i = 0, 1, ..., N - 1. We have an equal number of equations to solve them.

C3.8 Summary of equilibrium conditions for full model

This subsection summarises the equations of the labour market that define equilibrium and how they are integrated with the financial frictions model. These equations include N efficiency conditions that determine hours worked (33), the law of motion of workforce in each cohort (15), FOCs associated with the vacancy decision (37) and (40) for j = 0, ..., N - 1, derivative of the employment agency surplus with respect to the wage rate (61), the scaled agency surplus (59), the value function of a worker $V_{z^+,t}^i$ (62), derivative of the worker value function with respect to the wage rate (64), the growth adjustment term $G_{t,j}$ (39), the scaled value function for unemployed workers (58), the (suitably modified) resource constraint (67), equations that characterise productivity cut-off for job separations (52) and (56),

separations that characterise $\tilde{J}_{z^+,t}^{j}$ (57), the value of finding a job (21), the job finding rate (71), the probability of filling a vacancy (72), the matching function (22), the wage updating equation for cohorts that do not optimise (34), equation determining total employment (69), equation determining Ω_{t+j}^{j} (60), equation determining the hiring rate χ_{t}^{i} (14), equation determining the number of matches (the matching function) (68), the definition of total effective hours (70), equations defining \mathcal{M}_{t}^{j} (63), equations defining \mathcal{F}_{t}^{j} (32), equations defining \mathcal{E}_{t}^{i} (31), equations defining $\mathcal{G}_{t}^{j\prime}$ (50), equations defining $\mathcal{F}_{t}^{j\prime}$ (48).

The following additional endogenous variables are added to the list of endogenous variables in the financial frictions model:

 $l_{t}^{j}, \mathcal{E}_{t}^{j}, \mathcal{F}_{t}^{j}, \varsigma_{j,t}, \mathcal{M}_{t}^{j}, \bar{a}_{t}^{j}, \tilde{v}_{t}^{j}, G_{t,j}, Q_{t}, \Omega_{t+j}^{j}, J_{w,t}, w_{t}, J_{z^{+},t}, V_{z^{+},t}^{j}, U_{z^{+},t}, V_{w,t}^{0}, V_{z^{+},t}^{x}, f_{t}, m_{t}, v_{t}, \chi_{t}^{j}, \tilde{\pi}_{w,t}, L_{t}, \mathcal{G}_{t}^{j'}, \mathcal{F}_{t}^{j'} \text{ and } \tilde{J}_{z^{+},t}^{j}.$

We drop equations (4), (73), (74), (75), and (76) that determine wages from the financial frictions model:

$$w_{t} = \left[(1 - \xi_{w})(w_{t})^{\frac{\lambda_{w}}{1 - \lambda_{w}}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} w_{t-1} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}} \\ = \left[(1 - \xi_{w}) \left(\frac{1 - \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \right)^{\frac{1}{1 - \lambda_{w}}}}{1 - \xi_{w}} \right)^{\lambda_{w}} + \xi_{w} \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} w_{t-1} \right)^{\frac{\lambda_{w}}{1 - \lambda_{w}}} \right]^{\frac{1 - \lambda_{w}}{\lambda_{w}}}$$
(73),

$$F_{w,t} = \frac{\psi_{z^+,t}}{\lambda_w} w_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \frac{1-\tau^y}{1+\tau^w} + \beta \xi_w E_t \left(\frac{\overline{w}_{t+1}}{\overline{w}_t}\right) \left(\frac{\widetilde{\pi}_{w,t+1}}{\pi_{w,t+1}}\right)^{1+\frac{\lambda_w}{1-\lambda_w}} F_{w,t+1}$$
(74),

$$K_{w,t} = \zeta_t^h \left(w_t^{\frac{-\lambda_w}{1-\lambda_w}} h_t \right)^{1+\sigma_L} + \beta \xi_w E_t \left(\frac{\widetilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1}$$
(75)

The equations which describe the dynamic behaviour of the model are those of the financial frictions model plus those discussed in the employment frictions block. Finally, the resource constraint needs to be adjusted to include monitoring as well as recruitment (hiring) costs. Similarly, measured GDP is adjusted to exclude both monitoring costs and recruitment costs.

C4. Measurement equations

Below we report the measurement equations we use to link the model to the data. Our data series for inflation and interest rates are annualised in percentage terms, so we make the same transformation for the model variables, i.e. multiplying by 400:

$$\begin{split} R_t^{data} &= 400(R_t - 1) - \vartheta_1 400(R - 1) \\ R_t^{*,data} &= 400(R_t^* - 1) - \vartheta_1 400(R^* - 1) \\ \pi_t^{d,data} &= 400 \log \pi_t - \vartheta_1 400 \log \pi + \varepsilon_{\pi,t}^{me} \\ \pi_t^{c,data} &= 400 \log \pi_t^c - \vartheta_1 400 \log \pi^c + \varepsilon_{\pi^c,t}^{me} \\ \pi_t^{i,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \\ \pi_t^{*,data} &= 400 \log \pi_t^i - \vartheta_1 400 \log \pi^i + \varepsilon_{\pi^i,t}^{me} \end{split}$$

where $\varepsilon_{i,t}^{me}$ denotes the measurement error for the respective variables. In addition, $\vartheta_1 \in \{0,1\}$ allows us to handle demeaned and non-demeaned data. In particular, the data for interest rates and foreign inflation are not demeaned. The domestic inflation rates are demeaned.

We use non-demeaned first differences in total hours worked:

$$\Delta \log H_t^{data} = 100 \Delta \log H_t + \varepsilon_{H,t}^{me}$$

We use demeaned first-differenced data for the remaining variables. This implies setting $\vartheta_2 = 1$ below:

$$\begin{split} \Delta \log Y_{t}^{data} &= 100 \left(\log \mu_{z^{+},t} + \Delta \log \left[y_{t} - p_{t}^{i}a(u_{t}) \frac{\bar{k}_{t}}{\mu_{\psi,t}\mu_{z^{+},t}} - d_{t} - \frac{\kappa}{2} \sum_{j=0}^{N-1} (\tilde{v}_{t}^{j})^{2} (1 - \mathcal{F}_{t}^{j}) l_{t}^{j} \right] \\ &-\vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{y,t}^{me} \\ \Delta \log Y_{t}^{*,data} &= 100 (\log \mu_{z^{+},t} + \Delta \log y_{t}^{*}) - \vartheta_{2} 100 (\log \mu_{z^{+}}) \\ \Delta \log C_{t}^{data} &= 100 (\log \mu_{z^{+},t} + \Delta \log c_{t}) - \vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{c,t}^{me} \\ \Delta \log X_{t}^{data} &= 100 (\log \mu_{z^{+},t} + \Delta \log x_{t}) - \vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{x,t}^{me} \\ \Delta \log q_{t}^{data} &= 100 (\log \mu_{z^{+},t} + \Delta \log T ports_{t}) - \vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{M,t}^{me} \\ \Delta \log M_{t}^{data} &= 100 (\log \mu_{z^{+},t} + \Delta \log T ports_{t}) - \vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{M,t}^{me} \\ \end{bmatrix} \\ &= 100 \left[\log \mu_{z^{+},t} + \Delta \log \left(\frac{c_{t}^{m} (p_{t}^{m,c})^{\frac{\lambda_{m,c}}{1-\lambda_{m,c}}}}{+i_{t}^{m} (p_{t}^{m,t})^{\frac{\lambda_{m,i}}{1-\lambda_{m,k}}}} \right) \right] - \vartheta_{2} 100 (\log \mu_{z^{+}}) + \varepsilon_{M,t}^{me} \end{split}$$

 $\Delta \log I_t^{data} = 100[\log \mu_{z^+,t} + \log \mu_{\psi,t} + \Delta \log i_t] - \vartheta_2 100(\log \mu_{z^+} + \log \mu_{\psi}) + \varepsilon_{I,t}^{me}$ $\Delta \log G_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log g_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{g,t}^{me}.$

Note that neither measured GDP nor measured investment includes investment goods used for capital maintenance. To calculate measured GDP, we also exclude monitoring costs and recruitment costs.

The real wage is measured by the employment-weighted average Nash bargaining wage in the model:

$$w_t^{avg} = \frac{1}{L} \sum_{j=0}^{N-1} l_t^j G_{t-j,j} w_{t-j} \overline{w}_{t-j}.$$

Given this definition, the measurement equation for demeaned first-differenced wage is

$$\Delta \log(W_t/P_t)^{data} = 100\Delta \log \frac{\tilde{W}_t}{z_t^+ P_t} = 100(\log \mu_{z^+,t} + \Delta \log w_t^{avg}) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{W/P,t}^{me}.$$

Finally, we measure demeaned first-differenced net worth, interest rate spread and unemployment as follows:

$$\Delta \log N_t^{data} = 100(\log \mu_{z^+,t} + \Delta \log n_t) - \vartheta_2 100(\log \mu_{z^+}) + \varepsilon_{N,t}^{me}$$

$$\Delta \log Spread_t^{data} = 100\Delta \log(z_{t+1} - R_t) = 100\Delta \log\left(\frac{\overline{\omega}_{t+1}R_{t+1}^k}{1 - \frac{n_{t+1}}{p_{k',t}k_{t+1}}} - R_t\right) + \varepsilon_{Spread,t}^{me}$$

$$\Delta \log Unemp_t^{data} = 100\Delta \log(1 - L_t) + \varepsilon_{Unemp,t}^{me}$$

In a model with observed vacancies the vacancies are measured as the first difference of total vacancies.

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