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THE NATURAL RATE OF INTEREST: INFORMATION DERIVED FROM A SHADOW RATE MODEL

CONTENTS

ABBREVIATIONS

APP – asset purchase programme
ATSM – affine term structure of interest rates model
bps – basis points
CBPP – covered bond purchase programme
DSGE – dynamic stochastic general equilibrium (model)
ECB – European Central Bank
LB – lower bound on nominal interest rate
NRI – natural rate of interest
OIS – overnight indexed swap
REER – real effective exchange rate
TIPS – Treasury inflation protected securities
US – *United States* of America
ZLB – zero lower bound

ABSTRACT

The study proposes an estimation method of the natural rate of interest based on the shadow rate term structure of interest rates model and using information from nominal yields data. For the purpose of comparison and robustness check, different samples for the estimation of the natural rate of interest – three for the euro area and two for the US – are considered. The estimates based on all considered samples show a downturn trend in the estimated natural rates of interest for the euro area. However, since the beginning of 2013, this downward trend has levelled off. Compared to the results obtained by affine models, the shadow rate model produces lower estimates of natural rates of interest. From the beginning of 2013, the dynamics of estimated series of the US natural rate of interest closely follows the series produced by Laubach–Williams. However, before that the series are more divergent. In order to demonstrate the use of the natural rate of interest, we employ the estimated series of the natural rate of interest in the balance-approach version of the Taylor rule. The results imply that, at the end of the sample in July 2017, Taylor rule-suggested policy rates were in line with the actual ECB policy rates.

Keywords: natural rate of interest, term structure of interest rates, lower bound, non-linear Kalman filter

JEL codes: C24, C32, E43, E58, G12

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1. INTRODUCTION

In the aftermath of the crisis, the nominal interest rates in the euro area and many other advanced economies have stayed at the levels well-below historical averages. Some economists (Summers (2014), Williams (2016)) explain this phenomenon by a decline in the level of the natural (or, in other words, neutral or equilibrium) real rate of interest (NRI). Among factors that may have caused this decline are productivity decline, population aging, rising inequality, and the level of indebtedness. Williams (2015) defines the NRI as "the real federal funds rate consistent with the economy operating at its full potential once transitory shocks to aggregate supply or demand have abated". The NRI could be informative for policy makers as a: 1) reference point for interest rates set by central banks; 2) basis for long-term fiscal sustainability calculations; 3) benchmark for growth estimates; 4) measure for projecting long-term rates in financial markets.

The NRI, like potential output or equilibrium exchange rate, is unobservable; therefore, to estimate it we should rely on some econometric techniques. Four approaches are commonly used to estimate the NRI. One approach is to extract the natural rate as the long-run trend of a real rate time series (Hamilton et al. (2015)). Another approach is to use a small scale semi-structural model of the economy and with the help of the Kalman filter jointly extract both short-term shocks and long-term trends (Laubach and Williams (2003; 2016)). This approach allows for obtaining potential output, natural rate of unemployment, trend inflation and natural interest rate. The third approach uses a medium-scale DSGE model where the natural rate is the rate that would prevail if prices and wages were flexible (Barsky et al. (2014), Del Negro et al. (2017), Cúrdia et al. (2015)). The fourth approach is to use information from financial markets (from the yield curve) to estimate the level of the natural rate of interest.

The most popular and widely-used is the Laubach–Williams approach. However, it has some shortcomings. First of all, it uses only backward-looking macro-data. As a result, time series models estimated with historical data largely reflect the downward trend in the data. Another shortcoming is that the model is linear. Therefore, it ignores nonlinearities caused by the zero lower bound (ZLB). This drawback grows in importance as the period of central bank interest rates remaining close to zero becomes longer. In addition, macro-based estimates are distorted by revisions in output and inflation data that are not available in real time, i.e. when the estimate of the NRI is most needed. In addition, the review by Comunale and Striaukas (2017) presents a detailed list of drawbacks in the Laubach–Williams approach, and among those not mentioned are ignoring the REER, the role of risk, global factors, etc.

To avoid the problem of backward-looking data, Christensen and Rudebusch (2017) apply the fourth approach mentioned above, developing an Affine Term Structure of Interest Rates Model (ATSM). They use the TIPS data to utilise forward-looking information for the estimation of the NRI. Indeed, yield curve data are forward-looking as they contain the expected path of short-term interest rate (and the term premium as well). Christensen and Rudebusch (2017) assume that the longer-term expectations of the short-term real interest rate embedded in TIPS prices reflect financial market participants' views about the steady state of the real interest rate, i.e. the NRI. Another advantage of this measure of the NRI is that it can be obtained in real time at the same frequency as the underlying bond price data. However, this approach is difficult to apply to the euro market of inflation-linked financial instruments as this market is much less liquid and more fragmented than that of the US.

Brzoza-Brzezina and Kotłowski (2014) generalise the concept of the NRI by defining the natural yield curve. They use the dynamic Nelson–Siegel model and nominal yield curve data to estimate unobservable factors of the model; then, with the obtained factors in hand and adding macro-data, they estimate the factors of the natural yield curve.

This study also relates to the literature where very persistent factors are used in the term structure of interest rate model framework for a better explanation of the yield curve dynamics. Christensen et al. (2009) impose a near-unit-root restriction for the factor process in an ATSM framework to avoid small-sample estimation bias of less persistence in interest rate dynamics. Cieslak and Povala (2010) describe the dynamics of the yield curve in terms of two cycles with low and high frequencies. This allows them to explain the role of macroeconomic variables in bond risk premium. Dewachter and Lyrio (2006) propose a macro-finance model of the term structure of interest rates, assuming that long-run inflation and the real interest rate are latent martingale-difference processes. This allows interpreting these factors as long-run expectations of observable macroeconomic factors. Spencer (2008) develops a macro-finance model using a latent variable that follows the unit root process with stochastic volatility. Bauer and Rudebusch (2017) propose an approach that may refer to reverse-engineering. They take the existing estimates of NRI from macroeconomic literature and use them as an observable factor in the ATSM. They find that the proposed approach substantially increases the accuracy of long-range interest rate forecasts and improves the estimates of the term premium in long-term interest rates.

The present study uses the term structure of interest rates model to estimate the NRI. In contrast to Bauer and Rudebusch (2017), we construct the term structure of interest rates using the equilibrium nominal interest rate in a consistent way, i.e. obtained from the model. The contribution of this study is as follows. First, we develop and estimate a term structure of interest rates model with a persistent factor. The shadow rate is supposed to be equal to the sum of two unobservable factors. The first factor is transitional and disappears over time. The second factor is persistent and is assumed to be equal to the long-run short-term interest rate. We use the second order approximation (Ajevskis (2016)) and the extended Kalman filter to estimate model's coefficients and unobservable factors. The NRI is obtained by subtracting the expected inflation extracted from 5y5y inflation swap data from the average model nominal short rate over five-year period starting five years ahead.

Second, in contrast to Christensen and Rudebusch (2017) and Brzoza-Brzezina and Kotłowski (2014), the proposed model takes into account the nonlinearity caused by the lower bound (LB) of the nominal short interest rate, which is an important feature of the data in recent years. We also find that the estimates of the NRI are robust to the choice of samples of May 2008–July 2017 and July 2009–July 2017. The shadow rate model provides lower NRI estimates than the affine one. In order to demonstrate the use of natural rate of interest, we employ the estimated series NRI in the balance-approach version of Taylor rule. The results imply that, at the end of the sample in July 2017, Taylor rule-suggested policy rates were in line with the actual ECB policy rates. The minimum of the interest rate prescribed by the Taylor rule is –5% in January 2015, i.e. the time when the ECB announced its expanded APP.

2. MODEL

Laubach and Williams (2003) introduce several unit root processes into a semi-structural macro model, and with the help of the Kalman filter jointly extract both short-term shocks and long-term trends. One of the trends is treated as the NRI. The current study also exploits the idea that a persistent component relates to the equilibrium short-term interest rate but uses this feature in the framework of the term structure of interest rate model.

To construct the term structure of interest rates model for the estimation of the NRI, we choose a model specification with two unobservable factors rather than more conventional three factors. The reason is that two-factor specifications are more robust than three-factor ones, as is claimed by Krippner (2015a). Specifically, Krippner (2015a) illustrates the sensitivity of shadow rate for three-factor model with respect to the choice of LB parameter and different sample periods for estimation. Namely, he finds that using slightly different LB parameters and/or estimating with different samples of yield curve data produces shadow rate estimates with very different profiles and dynamics. Whereas the shadow estimates are relatively robust. Ichiue and Ueno (2013) also suggest to use two-factor models, arguing that "when interest rates are stuck at the ZLB and do not move much, a large part of the information required to identify the factors is missing, and thus the number of factors may have to be smaller than that when interest rates are far from zero." From this follows that three-factor models possibly overfit data and produce unrealistic estimates. In addition, Krippner (2015b; 2015c) shows that the two-factor affine Nelson–Siegel model provides the most parsimonious approximation to a generic affine term structure model with an arbitrary number of factors.

Hence, we assume that yields are driven by the vector $X_t = (X_t^1, X_t^2)$ of two unobservable factors that follow a first-order vector autoregressive process under real measure:

$$X_{t+1} = (1 - \Phi^{\mathbb{P}})\mu^{\mathbb{P}} + \Phi^{\mathbb{P}} X_t + \Sigma\varepsilon_{t+1},\ \varepsilon_{t+1} \sim N(0, I_N) \tag{1}$$

where Σ is a triangular matrix. The matrix of market price of risk Λ_t is given as an affine function of the factors

$$\Lambda_t = \lambda_0 + \lambda_1 X_t \qquad (2).$$

The dynamics of the factors under the risk-neutral measure is

$$X_{t+1}^1 = (1 - \Phi^{\mathbb{Q}})\mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} X_t^1 + \Sigma\varepsilon_{t+1},\ \varepsilon_{t+1} \sim N(0, I_N) \tag{3}$$

where $\mu^{\mathbb{Q}}$ and $\Phi^{\mathbb{Q}}$ satisfy

$$\mu^{\mathbb{Q}} = \mu^{\mathbb{P}} - \Sigma\lambda_0 \qquad (4),$$

$$\Phi^{\mathbb{Q}} = \Phi^{\mathbb{P}} - \Sigma\lambda_1 \qquad (5).$$

A shadow rate s_t is defined as

$$s_t = X_t^1 + X_t^2 \qquad (6),$$

and the observed short-term interest rate is the censored shadow rate

$$i_t = \max(s_t, i_j^{LB}) \quad (7)$$

where i_j^{LB}, $j = 1,2,\dots,k$ are LBs, and k is the number of lower bounds. Hence, our model framework allows for time-varying LBs of interest rates similar to Kortela (2016) and Lemke and Vladu (2016).

The bond price formula has the following form:

$$P_t^n = E_t^{\mathbb{Q}} \exp\left(\sum_{i=0}^{n-1} \max(s_{t+i}, i_j^{LB})\right) \quad (8),$$

which does not allow for affine representation of yields.

Observable yields are represented by

$$y_t^n = g_n\left(X_t, \theta, i_j^{LB}\right) = -\frac{1}{n}\ln\left\{E_t^{\mathbb{Q}} \exp\left(\sum_{i=0}^{n-1} \max(s_{t+i}, i_j^{LB})\right)\right\} + \eta_t \quad (9)$$

where η_t is the vector of measurement errors and θ is the vector of parameters.

Following Christeansen and Rudebusch (2015), we assume that

$$\Phi^{\mathbb{Q}} = \begin{pmatrix} 1 & 0 \\ 0 & \phi_{2,2} \end{pmatrix}$$

where $0 < \phi_{2,2} < 1$. Therefore, X_t^1 is the persistent factor, and it follows from equation (7) that over some sufficiently long time the main contribution in s_t will be from the factor X_t^1. We define the long-term nominal equilibrium rate of interest as

$$\lim_{\tau\to\infty} E_t s_{t+\tau} = X_t^1 = i_{l,t}^*.$$

Then, the long-term real rate of interest can be obtained from the Fischer equation:

$$r_{l,t}^* = i_{l,t}^* - \lim_{\tau\to\infty} E_t \pi_{t+\tau}.$$

The term $\lim_{\tau\to\infty} E_t \pi_{t+\tau}$ may be treated as a value of the long-term inflation. We can assign this value, for example, to the central bank inflation target, which is 2% in the case of the euro area. We define the NRI following Christensen and Rudebusch (2017) as the average real short-term rate over a five-year period starting five years ahead:

$$r_t^* = \frac{1}{60}\sum_{i=61}^{120} E_t r_{t+i} = \frac{1}{60}\left(\sum_{i=61}^{120} E_t i_{t+i} - \sum_{i=61}^{120} E_t \pi_{t+i}\right).$$

As a measure of the second term in the last formula we will use inflation-swap data.

3. DATA

For the euro area, the interest rate data are the end-of-themonth yields on the OIS for maturities of 3 and 6 months and 1, 2, 3, 4, 5, 7, 10 and 30 years from Bloomberg. We will consider three samples: one starting from July 2005, the second from May 2008, and the third from July 2009, all ending July 2017. The reason for the choice of the first sample is that the data for 7- and 10-year yields are available only from July 2005 (30-year yields are not available on this date, though). Bloomberg OIS rate of 30-year time to maturity data for the euro first became available from May 2008. The reason for including 30-year yields is that it makes the estimate of the natural nominal rate of interest smoother (less volatile). Other approaches mentioned above, for example, DSGE modelling, provide the estimation of the NRI that varies a lot over time, which is not acceptable if we consider the NRI as a long-run concept. Krippner (2015c) observes a similar phenomenon for shadow rate estimates; namely, he finds that within the shadow rate models the inclusion of 30-year yields in the sample lowers the volatility of estimates for shadow rate.

Regarding the third sample, in July 2009, the first ECB AIP was launched (the covered bond purchase programme; CBPP). This event can be treated as a structural break in the central bank policy. The choice of two last sample periods, which are relatively short, has an advantage of being less vulnerable to the Lucas critique. Indeed, when the short-term interest rate is stuck at a lower bound (the situation can be treated as a structural change in the economy), the use of an affine type model may be incorrect as shown in Krippner (2015c), and Christensen and Rudebusch (2015). For instance, let us consider a construction of the term structure of interest rates in a DSGE framework (see, e.g. Hördahl et al. (2006), and Rudebusch and Wu (2008)). In this case, the loadings for yields would be functions of deep parameters and the Taylor rule coefficients. When the short-term interest rate reaches the ZLB, the Taylor rule switches off, and thus its coefficients disappear in the yield loadings. Because of this, the loadings for yields must be different in the normal times and times when interest rates are constrained by the ZLB.

Overall, the two samples of May 2008–July 2017 and July 2009–July 2017 are relatively short, with 109 and 95 time observations respectively, thus they do not include a full business cycle. Nevertheless, this shortcoming is compensated by the forward-looking nature of the data and the absence of structural breaks during the time spans under consideration.

For the US, we use time series of yields of maturities of 1, 2, 3, 4, 5, 7, 10 and 30 years from Gürkaynak et al. (2007) for the two periods – one from January 2006 to May 2017, and the other from September 2008 to May 2017. The rate of 30-year time to maturity in Gürkaynak et al. (2007) data first became available from January 2006. The second sample starts with the onset of the crisis, i.e. September 2008. As a measure of inflation expectations, we use the end-of-the-month Bloomberg data on the euro area and US inflation swap for five-year period starting five years ahead (5y5y).

4. ESTIMATION

To guarantee the identification of parameters, we impose the following normalising restrictions: 1) $\mu^Q = 0$; 2) Φ^Q is a diagonal matrix; 3) Σ is lower triangular and 4) $\lambda_0^1 = 0$. In the shadow rate model, the effective LBs are assumed to equal the deposit facility rate set by the ECB when they are negative and zero otherwise in the case of the euro. In the case of the US, we assume that the effective LBs equal zero throughout the time span.

Overall, the model consists of 10 parameters, i.e. the vector (ϕ_{22}, σ_{11}, σ_{21}, σ_{22}, $\lambda_{0,1}$, $\lambda_{1,11}$, $\lambda_{1,11}$, $\lambda_{1,12}$, $\lambda_{1,22}$, σ_η). The state space model representation includes the transition dynamics given by equation (1) and the measurement system (9). As the measurement system is nonlinear, we apply the extended Kalman filter to estimate the model under the assumption that yields of all maturities are observed with measurement errors all having the same standard deviation σ_η. We use the Taylor series expansion of order two to approximate the bond prices (Ajevskis (2016)) for estimating both model's coefficients and unobservable factors. Linear models are estimated by the standard linear Kalman filter.

5. RESULTS

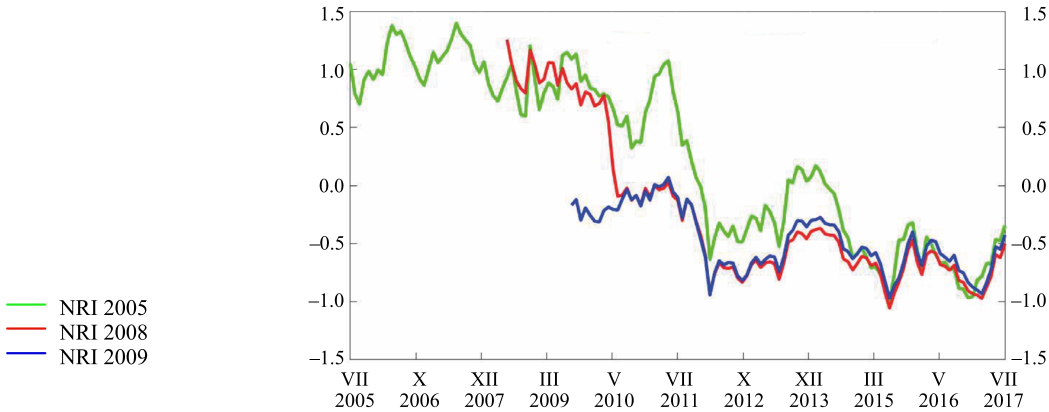
The estimated parameters of the model for the two samples are reported in Table. The estimates are robust to the samples except for few parameters. The measurement error for both samples has an estimated standard error of 2 bps, which indicates a good fit of the model.

Table. Kalman filter estimates for two periods of May 2008–July 2017 and July 2009–July 2017 (asymptotic standard errors are given in parenthesis)

Coefficients	2008	2009
$\phi_{1,1}$	0.9462	0.9457
	(0.1633)	(0.1633)
Σ_{11}	$-6 \cdot 10^{-6}$	$-6 \cdot 10^{-6}$
	$(1 \cdot 10^{-6})$	$(1 \cdot 10^{-6})$
Σ_{21}	$-1 \cdot 10^{-6}$	$-0.5 \cdot 10^{-5}$
	$(3 \cdot 10^{-7})$	$(2 \cdot 10^{-6})$
Σ_{22}	$2 \cdot 10^{-4}$	$1 \cdot 10^{-4}$
	$(3 \cdot 10^{-5})$	$(2 \cdot 10^{-5})$
$\lambda_{0,1}$	-0.239	0.559
	(0.070)	(0.001)
$\lambda_{1,11}$	327.0	-11.9
	(0.06)	(2.06)
$\lambda_{1,12}$	-2.93	-0.294
	(0.07)	(0.05)
$\lambda_{1,21}$	-243	-327
	(3.8)	(56.4)
$\lambda_{1,22}$	287	465
	(0.59)	(0.80)
σ_η	$2 \cdot 10^{-4}$	$2 \cdot 10^{-4}$
	$(5 \cdot 10^{-5})$	$(2 \cdot 10^{-5})$

Chart 1 shows the estimated NRIs for the three samples that we denote NRI 2005, NRI 2008 and NRI 2009. All the NRIs follow a slight downturn trend over the samples. However, this trend seems to have disappeared since the beginning of 2012. The NRI 2008 and NRI 2009 estimates are very close to each other starting from June 2010. Before this date, the NRI 2008 estimate is closer to the NRI 2005 than to the NRI 2009. The NRI 2005 is noticeably higher than NRI 2008 and NRI 2009 over almost the whole sample, and it is also much more volatile. This may be explained by a possible bias in the estimate due to structural changes in the monetary policy, i.e. APP, forward guidance and the negative ECB deposit rate. The NRI estimates are more robust over shorter sample periods. For this reason, further on we use two shorter samples from May 2008 to July 2017 and from July 2009 to July 2017. However, all three NRIs are very close to each other starting with the second half of 2014. At the end of the sample, i.e. July 2017, all three NRIs are negative and range from –0.35% to –0.50%.

Chart 1
Euro area NRI for three samples



Next, we explore the effect of effective LB on the estimate of the NRI. We compare the NRI estimated by the affine and shadow rate models. The affine model has the same structure as the shadow rate model except for the constraint (equation (7)) and the fact that the short-term rate is defined by equation (6). Chart 2 shows the NRI estimated using the affine and shadow rate models for the samples starting from May 2008 and July 2009. The NRI estimated by the affine model exceeds that obtained by the shadow rate model over the whole sample period. It is not surprising, since in the case of affine model, the sum of factors is the short-term interest rate that is bounded by the LB, whereas for the shadow rate model, the sum of the factors is the shadow rate that can also be negative, i.e. lower than the short-term interest rate. When the shadow rate becomes negative, both factors decrease. Again, we see that at the end of the sample, the difference between natural rates of interest of different samples and different models is very small. Also, in the case of affine model, we see that the estimate of the factor X_t^1 is quite robust to the sample chosen.

Chart 2
Euro area NRI for shadow rate and affine models

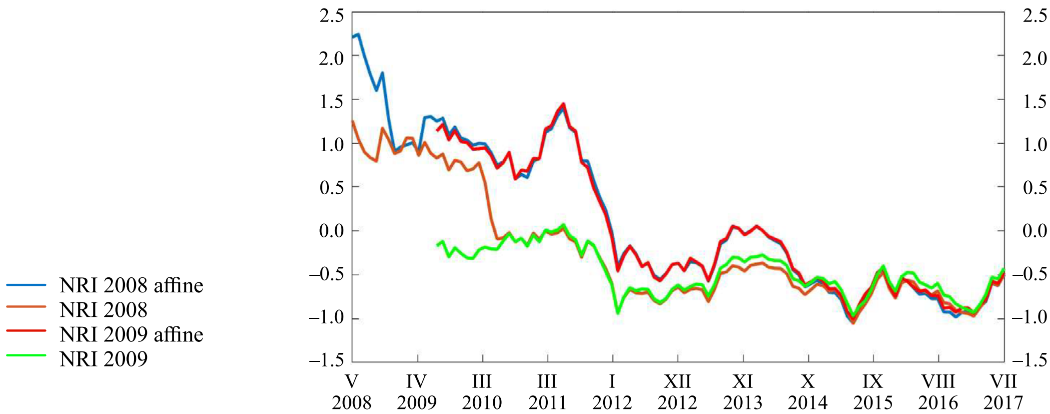
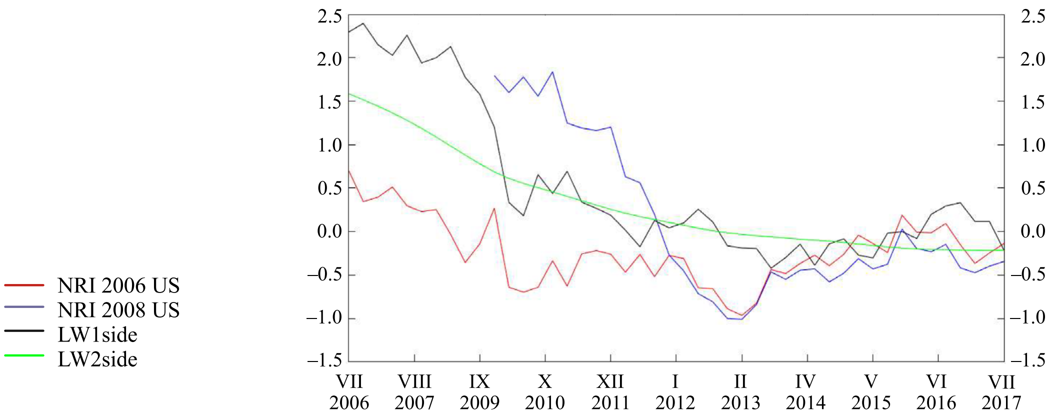


Chart 3 shows the estimate of the NRI based on the US data. For comparison purposes, we also add the Laubach–Williams one-side and two-side filtered estimates of the NRI (LW1side and LW2side) in Chart 3[1]. As the Laubach–Williams estimation is based on quarterly data, we convert the estimates from a monthly to quarterly frequency,

[1] The Laubach–Williams estimates are taken from the web-site http://www.frbsf.org/economic-research/economists/LWreplication.zip.

choosing the last observation for each quarter. All estimates are characterised by a downturn trend. However, similar to the euro area, this trend seems to have disappeared since the end of 2012. At the beginning of both samples, the corresponding NRI estimates of the shadow rate model differ from those of the Laubach–Williams model. In the first quarter of 2006, the differences between NRI 2006 and LW1side and LW2side were 1.6% and 0.8% respectively. In the third quarter of 2008, the differences between NRI 2008 and LW1side and LW2side were 0.5% and 1.1% respectively. After the second quarter of 2013, however, differences between the Laubach–Williams estimates and that of the shadow rate are minor. For example, at the end of the sample in the second quarter of 2017, the values of NRI for LW1side, LW2side, NRI 2006 and NRI 2008 are 0.22%, 0.22%, 0.14% and 0.34% respectively.

Chart 3
US NRI for shadow rate and Laubach–Williams models



6. POLICY RATE IMPLIED BY TAYLOR RULE

From the monetary policy perspective, the NRI serves as a reference point for operationalising the nominal policy short-term rate via the Taylor rule

$$i_t = r_t^* + \pi^* + 0.5(\pi_t - \pi^*) + 0.5(y_t - y_t^*) \tag{10}$$

where i_t represents the value of the policy rate prescribed by the rule, r_t^* is the NRI, π_t is price inflation, $\pi^* = 2$ is the inflation rate at its 2 percent longer-run objective, y_t is output, and y_t^* is potential output. In his pioneering work, Taylor (1993) assumes that r_t^* is constant and equal to 2%, i.e. the average real trend GDP growth between 1984 and 1992. Yellen (2015), instead, allows for the time-varying natural real rate based on econometric estimates, claiming that it is a more realistic assumption in the current low nominal interest rate environment. Numerous estimates by various authors have confirmed the last point in recent years. Nonetheless, Taylor and Wieland (2016) challenge the approach based on the estimate of the NRI, arguing that it is not accurate and is biased due to the omitted variable and equation problem.

However, we evaluate the policy short-term rate using a type of the Taylor–Yellen rule, called the balance-approach rule (BGFRS (2017)), where the output gap is approximated by the unemployment gap using the Okun's law:

$$i_t = r_t^* + \pi_t + 0.5(\pi_t - \pi^*) + 2(u_t^* - u_t) \tag{11}$$

where u_t is the unemployment rate, u_t^* is the rate of unemployment in the longer run.

Chart 4 illustrates the short-term policy rate prescribed by the balance-approach rule with the rate r_t^* substituted by the NRI 2008.[2] Chart 4 shows that since the beginning 2012, the nominal short-term interest rate prescribed by the rule had become negative. In January 2015, the policy rate reached its minimum of less than –5%. It is hardly possible for a central bank to keep such a negative policy rate. Probably, this was the reason why the ECB announced its expanded APP at exactly that time, i.e. when the monetary accommodation was needed most for the euro area economy. From the middle of 2016, the rate is gradually trending upwards, with the last estimate in July 2017 at 0.15% and 0.22% for NRI 2008 and NRI 2009 respectively. This rate is very close to the current ECB's policy rate (fixed rate) of 0%.

Chart 4
Policy nominal interest rates prescribed by the balance-approach rule

7. CONCLUSIONS

This study estimates the equilibrium nominal interest rate, using the shadow rate term structure of interest rate model and nominal yield data for the euro area and the US. Using the Fisher equation and inflation swap data, we then estimate the real NRI. In the case of the euro area, three alternative sample periods of estimation, starting from July 2005, May 2008 and July 2009, are considered. For all samples, a slight downturn trend in natural rates of interest can be observed. This trend dissipates in 2013, though. The NRI estimates of the shadow rate model are lower than those of the affine model. However, the difference between the estimates is minor at the end of the sample. Approximately from the third quarter of 2013, the estimates based on the US data are close to the estimates obtained by the Laubach–Williams (2003) method.

Knowing the estimate of the NRI could give central banks additional information that helps them make decisions on policy rates. We estimate the short-term nominal interest rate suggested by a version of the Taylor rule, called the balance-approach rule. While the interest rate prescribed by the Taylor rule achieved its minimum in January 2015, i.e. at the time when the ECB announced its expanded APP, the estimates for the last period of the sample, i.e. July 2017, are 15 bps and 22 bps, i.e. close to the ECB's policy rate.

[2] The difference between short-term policy rates based on the estimates of the NRI 2008 and NRI 2009 is almost negligible.

BIBLIOGRAPHY

AJEVSKIS, Viktors (2016). *A Term Structure of Interest Rates Model with Zero Lower Bound and the European Central Bank's Non-Standard Monetary Policy Measures*. Latvijas Banka Working Paper, No. 2/2016. 18 p.

BARSKY, Robert, JUSTINIANO, Alejandro, MELOSI, Leonardo (2014). The Natural Rate of Interest and Its Usefulness for Monetary Policy. *American Economic Review*, vol. 104, No. 5, May 2014, pp. 37–43.

BAUER, Michael D., RUDEBUSCH, Glenn D. (2017). *Interest Rates under Falling Stars*. Federal Reserve Bank of San Francisco Working Paper, No. 2017-16, November 2017.

BGFRS (2017). *Board of Governors of the Federal Reserve System Monetary Policy Report*, 7 July 2017. 67 p.

BRZOZA-BRZEZINA, Michał, KOTŁOWSKI, Jacek (2014). Measuring the Natural Yield Curve. *Applied Economics*, vol. 46, issue 17, pp. 2052–2065.

CHRISTENSEN, Jens H. E., DIEBOLD, Francis X., RUDEBUSCH, Glenn D. (2009). An Arbitrage-Free Generalized Nelson–Siegel Term Structure Model. *Econometrics Journal*, Royal Economic Society, vol. 12, No. 3, November 2009, pp. 33–64.

CHRISTENSEN, Jens H. E., RUDEBUSCH, Glenn D. (2015). Estimating Shadow-Rate Term Structure Models with Near-Zero Yields. *Journal of Financial Econometrics*, vol. 13, No. 2, pp. 226–259.

CHRISTENSEN, Jens H. E., RUDEBUSCH, Glenn D. (2017). *A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt*. Federal Reserve Bank of San Francisco Working Paper, No. 2017-07, July 2017.

CIESLAK, Anna, POVALA, Pavol (2015). Expected Returns in Treasury Bonds. *The Review of Financial Studies*, vol. 28, issue 10, pp. 2859–2901.

COMUNALE, Mariarosaria, STRIAUKAS, Jonas (2017). *Unconventional Monetary Policy: Interest Rates and Low Inflation: A Review of Literature and Methods*. CORE Louvain DP, No. 2017/26, September 2017. 54 p.

CÚRDIA, Vasco, FERRERO, Andrea, NG, Ging Cee, TAMBALOTTI, Andrea (2015). Has U.S. Monetary Policy Tracked the Efficient Interest Rate? *Journal of Monetary Economics*, vol. 70, issue C, pp. 72–83.

DEL NEGRO, Marco, GIANNONE, Domenico, GIANNONI, Marc, TAMBALOTTI, Andrea (2017). *Safety, Liquidity, and the Natural Rate of Interest*. Brookings Papers on Economic Activity, BPEA Conference Drafts, 23–24 March, 2017. 77 p.

DEWACHTER, Hans, LYRIO, Marco (2006). Macro Factors and the Term Structure of Interest Rates. *Journal of Money, Credit and Banking*, vol. 38, issue 1, pp. 119–140.

GÜRKAYNAK, Refet, SACK, Brian, WRIGHT, Jonathan (2007). The U.S. Treasury Yield Curve: 1961 to the Present. *Journal of Monetary Economics*, vol. 54, issue 8, pp. 2291–2304.

HAMILTON, James D., HARRIS, Ethan S., HATZIUS, Jan, WEST, Kenneth D. (2015). *The Equilibrium Real Funds Rate: Past, Present and Future*. NBER Working Paper, No. 21476, August 2015. 82 p.

HÖRDAHL, Peter, TRISTANI, Oreste, VESTIN, David (2006). A Joint Econometric Model of Macroeconomic and Term-Structure Dynamics. *Journal of Econometrics*, vol. 131, issue 1–2, pp. 405–444.

ICHIUE, Hibiki, UENO, Yoichi (2013). *Estimating Term Premia at the Zero Bound: An Analysis of Japanese, US, and UK Yields*. Bank of Japan Working Paper, No. 13-E-8, May 2013. 30 p.

KORTELA, Tomi (2016). *A Shadow Rate Model with Time-Varying Lower Bound of Interest Rates*. Bank of Finland Research Discussion Paper, No. 19/2016. 54 p.

KRIPPNER, Leo (2015a). *A Comment on Wu and Xia (2015), and the Case for Two-Factor Shadow Short Rates*. CAMA Working Paper, No. 48/2015, Australian National University, December 2015. 38 p.

KRIPPNER, Leo (2015b). A Theoretical Foundation for the Nelson–Siegel Class of Yield Curve Models. *Journal of Applied Econometrics*, vol. 30, No. 1, January 2015, pp. 97–118.

KRIPPNER, Leo (2015c). *Zero Lower Bound Term Structure Modeling: A Practitioner's Guide*. Palgrave Macmillan. 409 p.

LAUBACH, Thomas, WILLIAMS, John C. (2003). Measuring the Natural Rate of Interest. *The Review of Economics and Statistics*, MIT Press, vol. 85, issue 4, November 2003, pp. 1063–1070.

LAUBACH, Thomas, WILLIAMS, John C. (2016). Measuring the Natural Rate of Interest Redux. *Business Economics*, vol. 51, No. 2, pp. 57–67.

LEMKE, Wolfgang, VLADU, Andreea L. (2016). *Below the Zero Lower Bound – A Shadow-Rate Term Structure Model for the Euro Area*. Deutsche Bundesbank Discussion Paper, No. 32/2016. 50 p.

RUDEBUSCH, Glenn D., WU, Tao (2008). A Macro-Finance Model of the Term Structure, Monetary Policy and the Economy. *Economic Journal*, No. 118, July 2008, pp. 906–926.

SPENCER, Peter D. (2008). Stochastic Volatility in a Macro-Finance Model of the U.S. Term Structure of Interest Rates 1961–2004. *Journal of Money, Credit and Banking*, vol. 40, issue 6, September 2008, pp. 1177–1215.

SUMMERS, Lawrence H. (2014). U.S. Economic Prospects: Secular Stagnation, Hysteresis, and the Zero Lower Bound. *Business Economics*, vol. 49, No. 2, pp. 65–73.

TAYLOR, John B. (1993). *Discretion versus Policy Rules in Practice*. Carnegie–Rochester Conference Series on Public Policy 39, pp. 195–214.

TAYLOR, John B., WIELAND, Volker (2016). *Finding the Equilibrium Real Interest Rate in a Fog of Policy Deviations*. CEPR Discussion Paper, No. DP11264, May 2016. 22 p.

WILLIAMS, John C. (2015). The Decline in the Natural Rate of Interest. *Business Economics*, vol. 50, issue 2, April 2015, pp. 57–60.

WILLIAMS, John C. (2016). *Monetary Policy in a Low R-star World*. Federal Reserve Bank of San Francisco Economic Letter No. 2016-23, 15 August 2016.

YELLEN, Janet L. (2015). *Normalizing Monetary Policy: Prospects and Perspectives*. Remarks at the conference on New Normal Monetary Policy, Federal Reserve Bank of San Francisco. Board of Governors of the Federal Reserve System (US).